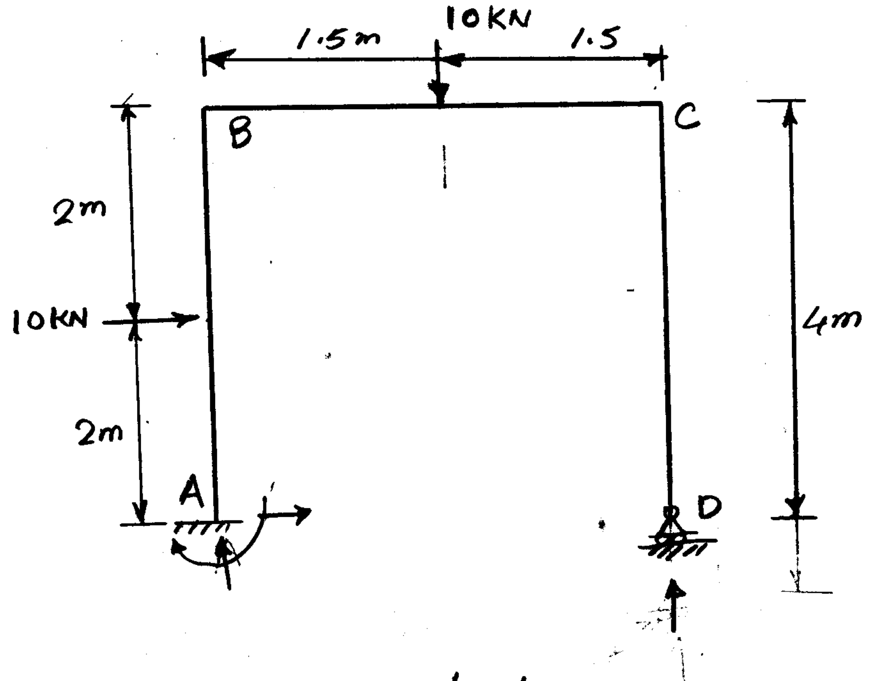
Compatibility Method of Analysis

Example Problem – Frame Structure

Analyze the Frame structure shown below using the Compatibility Method of the Flexibility Method.



EI Constant

Solution

Degree of Indeterminancy ?

Unknown Rections – 3 Statics Equations

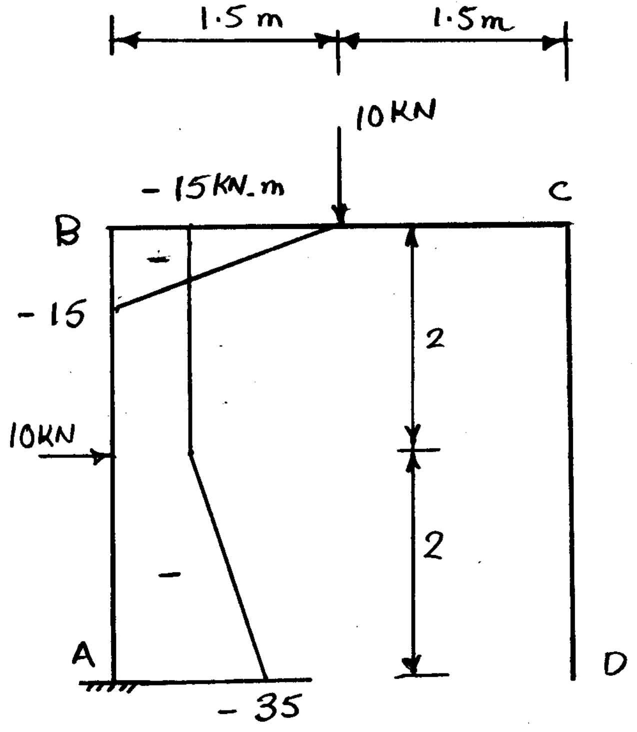
3 Reactions @ A + 1 Reaction @ D – 3

4 – 3 = 1 Degree of Indeterminancy.

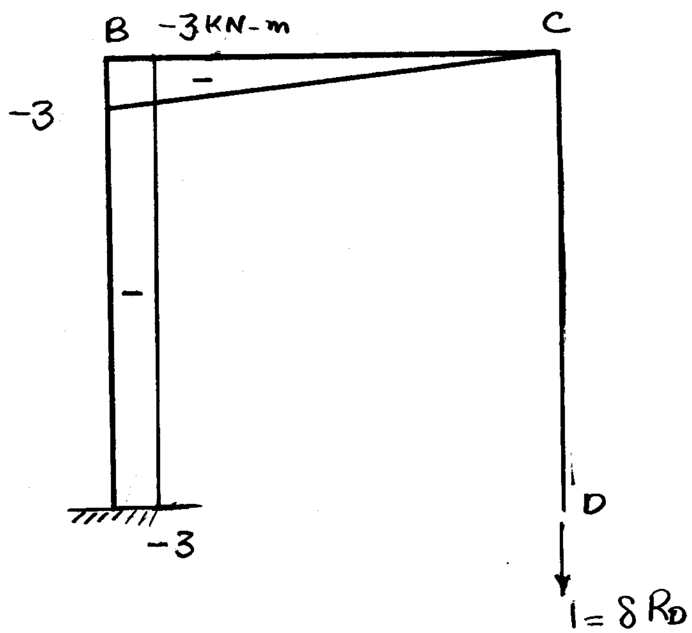
Flexibility/ Compatibility Method

Example Problem

Primary Determinate Structure



Actual BM
Primary System



Virtual BM
Primary Structure

Vertical Displ. @ Pt "D" due to applied Loading

$$1.\quad \Delta D_1 = \sum_{j=1}^{m} \int_{\ell} \left(\frac{M_D M_V}{EI} \right)_j dx$$

$$= \int_{1.5}^{3.0} \frac{-10(x-1.5)\cdot(-x)}{EI}\,dx + \int_{0}^{2} \frac{(-15)(-3)}{EI}\,dx$$

$$+ \int_{0}^{2} -\frac{(15+10x)\cdot(-3)}{EI}\,dx$$

$$= \frac{10}{EI}\left[\frac{x^3}{3} - \frac{1.5x^2}{2}\right]_{1.5}^{3.0} + \frac{45}{EI}\left[x\right]_0^2 + \frac{15}{EI}\left[3x + \frac{2x^2}{2}\right]_0^2$$

Example Problem

$$\Delta D_1 = \frac{10}{EI}\left[\frac{3^3 - 1.5^3}{3} - \frac{1.5}{2}\left(3^2 - 1.5^2\right)\right]$$

$$+ \frac{45}{EI} \times 2 + \frac{15}{EI}\left[3 \times 2 + \frac{2 \times 2^2}{2}\right]$$

$$= \frac{28.13 + 90 + 150}{EI} = \frac{268.13}{EI} \downarrow \text{ Downwards}$$

Vertical Displ @ D due to Unit Vertical load.

$$1.\ \delta D_1 = \sum_{j=1}^{m} \int_0^l \left(\frac{M_v^2}{EI}\, dx\right)_j$$

$$= \int_0^3 \frac{(-x)^2}{EI} dx + \int_0^4 \frac{(-3)^2}{EI} dx$$

$$= \left[\frac{x^3}{3EI}\right]_0^3 + \left[\frac{9}{EI} x\right]_0^4$$

$$\delta D_1 = \frac{3^3}{3EI} + \frac{9 \times 4}{EI} = \frac{45}{EI} \downarrow \text{ Downwards.}$$

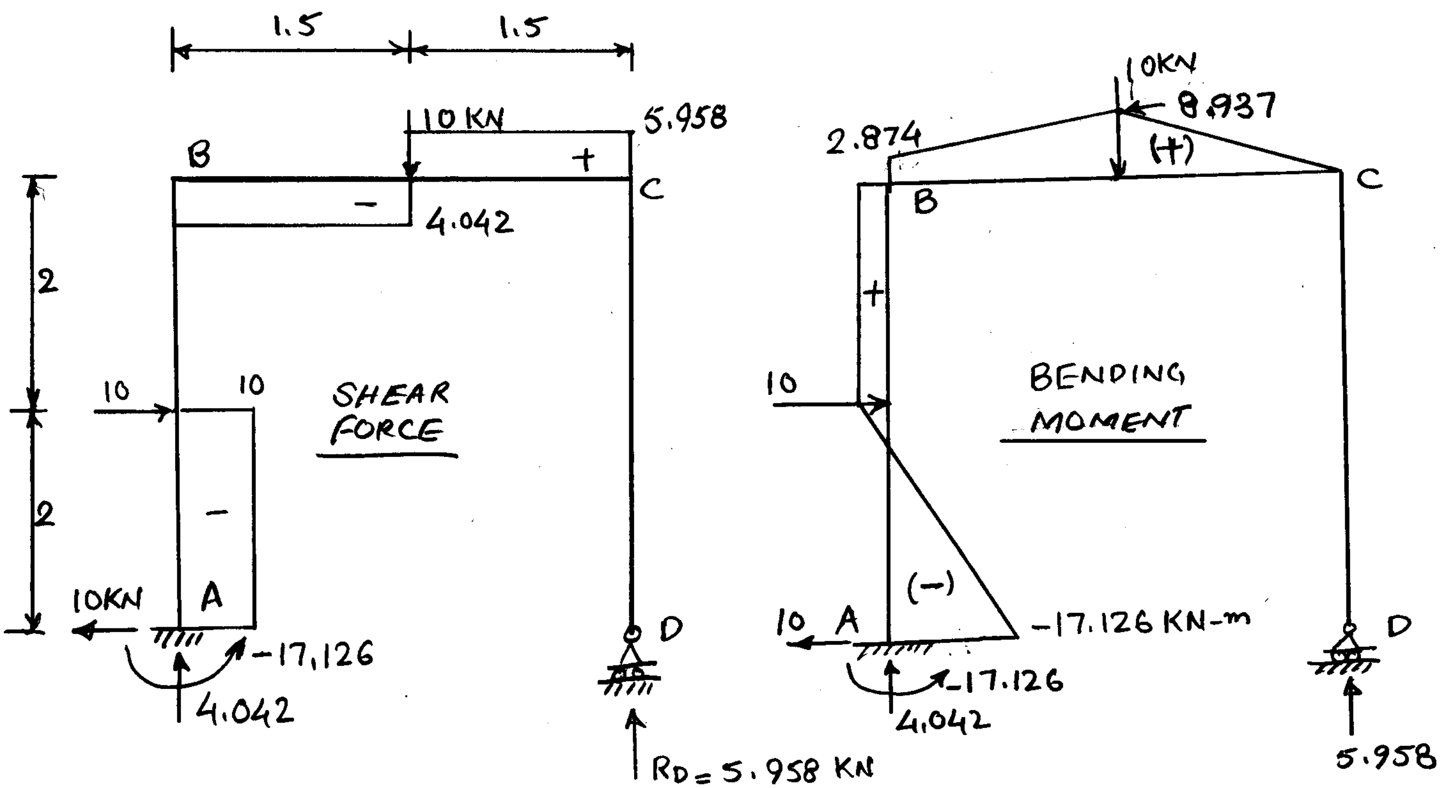
Compatibility Condition

$$\Delta D_1 + R_B\, \delta D_1 = 0$$

$$\Rightarrow R_B = \frac{-\Delta D_1}{\delta D_1} = -\frac{268.13}{EI} \times \frac{EI}{45} = -5.958 \text{ KN} \uparrow \text{ Upwards.}$$

Compatibility Method of Analysis

Example Problem - Frame Structure



$$-5.958 \times 3 + 10 \times 1.5 + 10 \times 2 + M_A = 0$$

$$\Rightarrow M_A = -17.126 \text{ KN-m}.$$

Trusses can have the following types of indeterminancies associted with them:

(a) External Indetetorminancy:
Support Reactions are more than that can be solved for by statics alone

(b) Internal Indeterminancy:
When the number of truss members are more than what can be determined statically.

(c) Combination of the above:

Degree of Indeterminancy of a truss $= m + r - 2n$

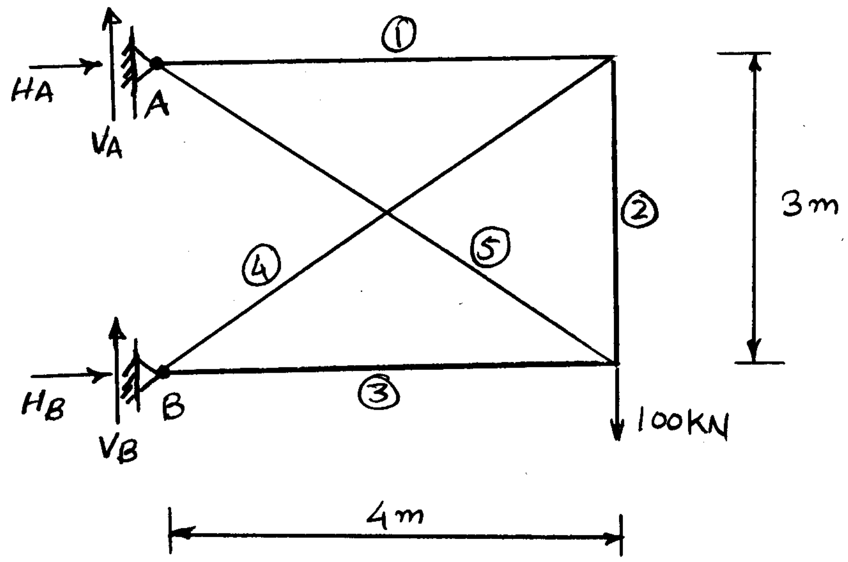
Where, $m =$ No of Members
$r =$ No. of Reactions
$n =$ No. of Joints

- In case of External indeterminancy, the redundant support is removed and displacement at the support computed. As there is actually no displacement at the support, the support reaction should be such that the displacement is reduced to zero.
- In Case of Internal Indeterminancy, the indeterminancy is removed by making a "cut" in the redundant members. As a result a gap develops in the redundant members. By applying suitable forces in the redundant members the cuts are closed, yielding forces in the redundant members.

Compatibility Method of Analysis

Example - Truss Problem

Problem



Analysize the Truss shown above using compatibility method of analysis.

Solution

By inspection the truss is externally indeterminate by degree one as we have 4 unknown reactions and 3 equations of statics

Total Degree of Indeterminancy $= I = m + r - 2n$

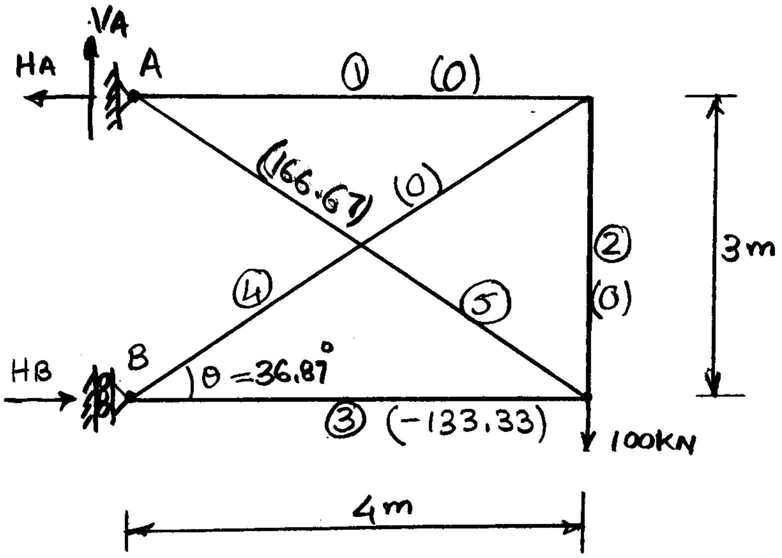
m = No of members
r = No. of Reactions
n = No. of Joints

$$\Rightarrow I = 5 + 4 - 2(4)$$
$$= 5 + 4 - 8 = 1$$

$\Rightarrow$ Externally Indeterminate by degree 1

Compatibility Method of Analysis

Example Problem – Truss



Primary Structure

External Redundancy is removed by making the support at pt B a roller. Reactions can then be determined.

Taking moments @ B

$$100 \times 4 - HA \times 3 = 0$$

$$\Rightarrow \quad HA = \frac{100 \times 4}{3} = 133.33 \text{ KN} \leftarrow$$

$$HB = \quad = 133.33 \text{ KN} \rightarrow$$

$$VA = \quad = 100 \text{ KN} \uparrow$$

Forces in the Truss Members

$$F_4 \sin\theta = 0 \quad \Rightarrow \quad F_4 = 0$$

$$F_3 + HB = 0 \quad \Rightarrow \quad F_3 = -HB = -133.33 \text{ KN (Comp)}$$

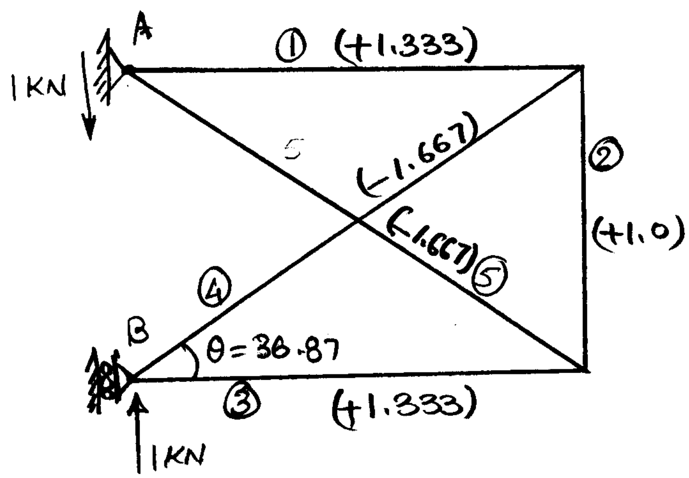
$$F_5 \sin\theta = VA = 100 \Rightarrow F_5 = \frac{100}{\sin 36.87} = 166.67 \text{ KN (Tens)}$$

As $F_4 = 0$ $\quad \Rightarrow F_1 = 0$

$\Rightarrow F_2 = 0$

Compatibility Method of Analysis

Example Problem – Truss



Unit Load @ B

To find Displacement at Pt B we apply a unit load @ Pt B

Forces in Truss due to Unit Load

$f_4 \sin 36.87 + 1 = 0 \Rightarrow f_4 = \dfrac{-1}{\sin 36.87} = -1.667$ (Comp)

$f_3 + f_4 \cos 36.87 = 0 \Rightarrow f_3 = -f_4 \cos 36.87 = +1.333$ (Tens)

$f_5 = f_4$ by inspection $\Rightarrow f_5 = f_4 = -1.667$

$f_1 = f_3$ " " $\Rightarrow f_1 = f_3 = +1.333$

$f_2 + f_4 \sin\theta = 0 \Rightarrow f_2 = -f_4 \sin 36.87 = +1.0$ KN(Tens)

Compatibility Method of Analysis

Example - Truss Problem

Next we compute the Displacement @ pt "B" in the Primary Structure using the "unit load method"

$$1.\ \Delta_B = \sum_{j=1}^{m} \frac{F \cdot f \cdot L}{AE}$$

F = Forces in the Primary Structure due to Real applied Loads

f = Forces in the Primary Structure due to Ficticious unit load

Computations are carried out in Tabular Format below:

Member	F (KN)	L (m)	f (KN)	FfL	f^2L	f.RB (KN)	F+f.RB (KN)
1	0	4.0	+1.333	0	+7.108	+62.204	+62.204
2	0	3.0	+1.0	0	+3.0	+46.665	+46.665
3	-133.33	4.0	+1.333	+710.92	+7.108	+62.204	-71.126
4	0	5.0	-1.667	0	+13.894	-77.791	-77.791
5	+166.67	5.0	-1.667	-1389.2	+13.894	-77.791	+88.879
				-2100.12	+45.004		

$$1.\ \Delta_B = \sum_{j=1}^{m} \frac{FfL}{AE} = \frac{-2100.12}{AE} \downarrow \text{(Downwards)}$$

Deflection δ_B due to Unit Load applied at pt B

$$\delta_B = \sum_{j=1}^{m} \frac{f^2 L}{AE}$$

Deflection Δ'_B due to Reaction "RB" at Pt "B"

$$\Delta'_B = R_B \cdot \delta_B = R_B \times \frac{45.004}{AE} = \frac{45.004\, R_B}{AE}$$

Compatibility Method of Analysis

Example - Truss Problem

Writing Compatibility Condition:

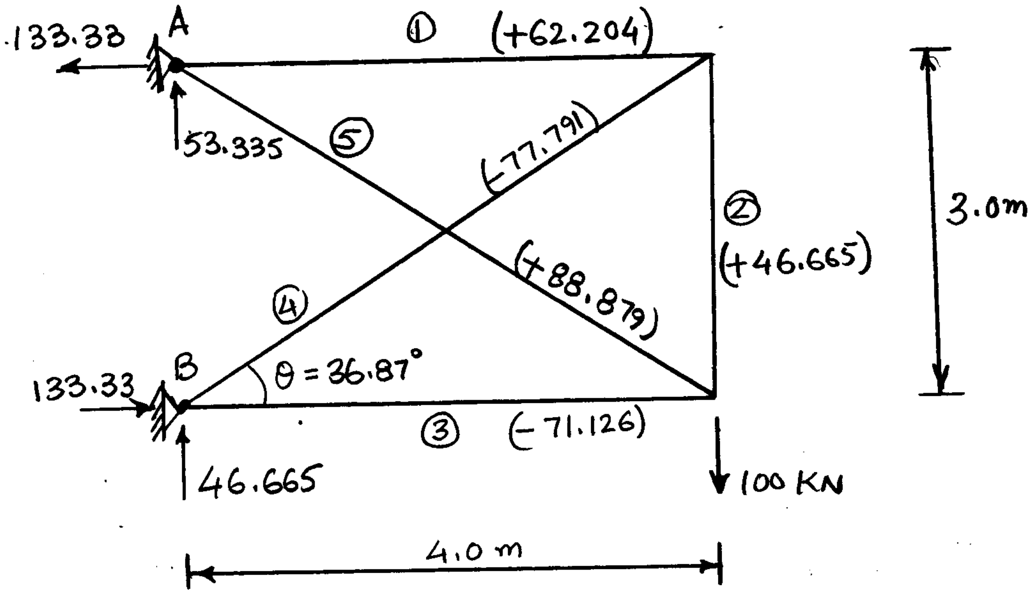
$$\Delta_B \downarrow + \Delta_B' \uparrow = 0$$

$$\frac{-2100.12}{AE} + \frac{45.004\, R_B}{AE} = 0$$

$$\Rightarrow R_B = \frac{2100.12}{45.004} = 46.665 \text{ KN} \uparrow \text{(upwards)}$$

The Forces in the Truss members due to RB can be computed by Taking the product "f. RB".

The Final Forces in the Truss can be computed by Taking the Sum "f. RB + F". These computions are shown as the last 2 columns of the Table

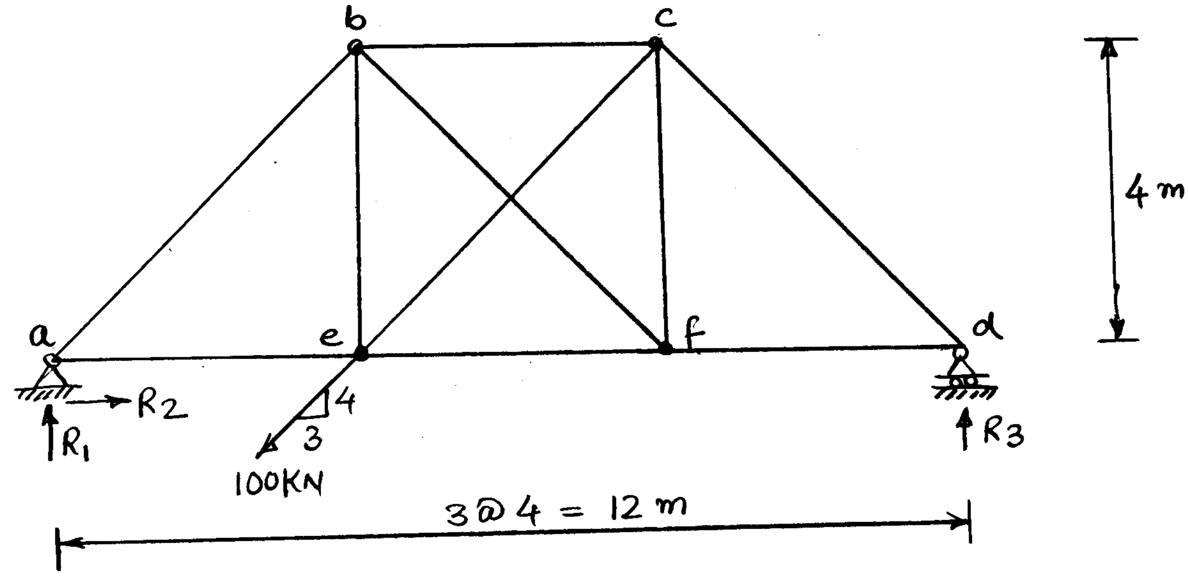


FINAL FORCES AND REACTIONS

Example Problem - Truss internally Indeterminate

Problem:-

Determine the member Forces. EA Constant.



Indeterminancy: ?

External Reactions $r_a = 3$

Equations of Statics $r = 3$

$\Rightarrow$ $r_a = r$, $3 = 3$ $\Rightarrow$ Statically Determinate & Stable.

No of Members $m = 10$

No of Joints $n = 6$

No of Reactions $r = 3$

Internal Indeterminancy $I = m + r - 2n$

$$= 10 + 3 - 2\times 6$$

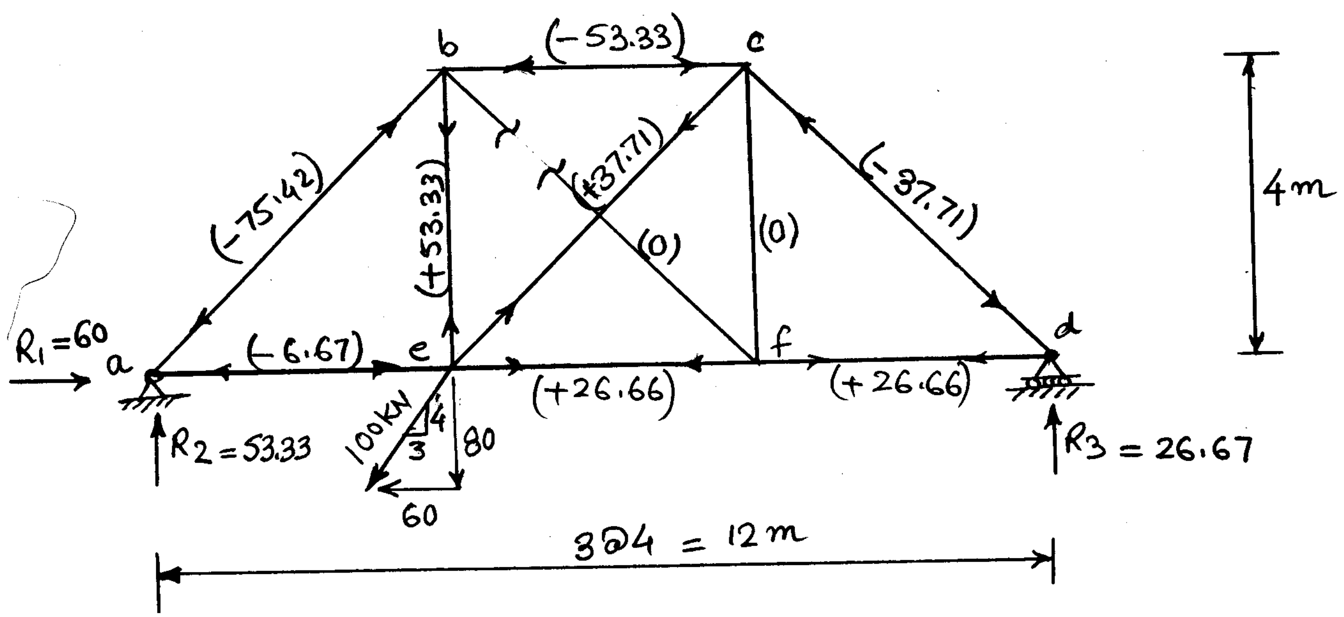
$$= 13 - 12 = 1$$

$\Rightarrow$ Structure Internally Indeteterminate by degree 1.

Compatibility Method of Analysis

Example Problem – Truss Internally Indeterminate

Member bf is selected as redundant member



Primary Structure

$$f_{cd} = -\frac{26.67}{\sin 45} = -37.71 \text{ (Comp)}$$

$$f_{fd} = 37.71 \cos 45^\circ = +26.66 \text{ (Tens)}$$

$$f_{ab} = -\frac{53.33}{\sin 45^\circ} = -75.42 \text{ (Comp)}$$

$$f_{bc} = f_{ab} \cdot \cos 45 = -75.42 \times \cos 45 = -53.33 \text{ (Comp)}$$

$$f_{be} = -f_{ab} \sin 45 = 75.42 \times \sin 45 = +53.33 \text{ (Tens)}$$

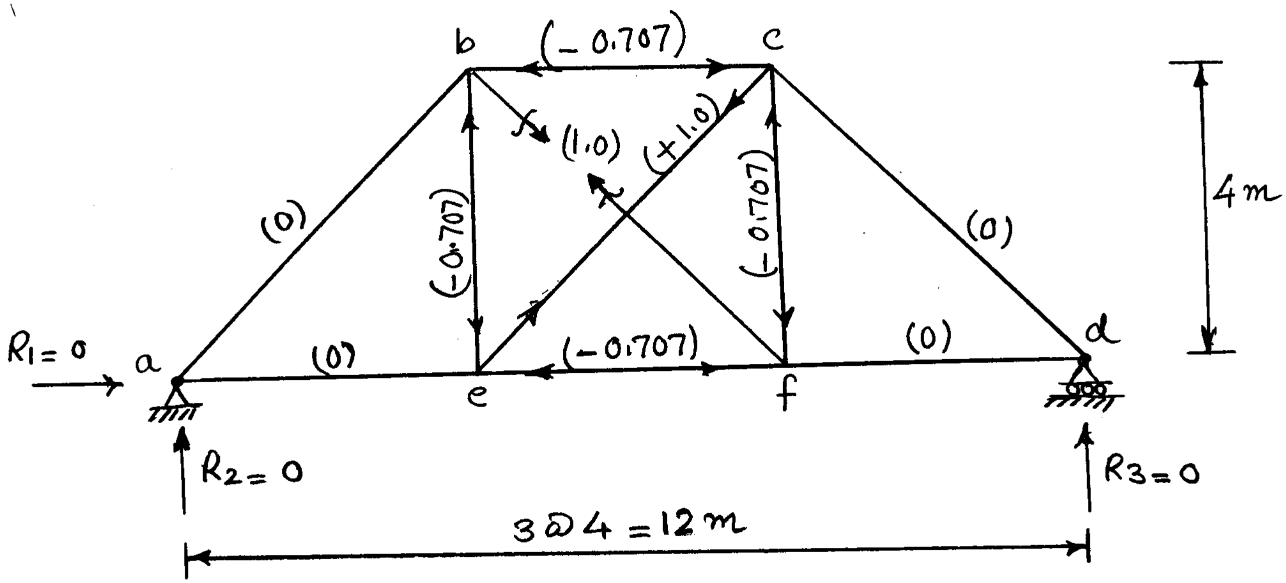
$$f_{ae} = -f_{ab} \cos 45 - 60 = 75.42 \cos 45 - 60 = -6.67 \text{ (Comp)}$$

$$f_{cd} \sin 45 + f_{ce} \sin 45 = 0$$

$$\Rightarrow f_{ce} = -f_{cd} = -(-37.71) = +37.71 \text{ (Tens)}$$

Compatibility Method of Analysis

Example Problem - Truss - Internally Indeterminate



$f_{ab} = f_{ae} = f_{fd} = f_{cd} = 0$ by inspection.

$f_{bc} = -1 \times \cos 45 = -0.707$ (Comp)

$f_{be} = -1 \times \cos 45 = -0.707$ (Comp)

$f_{cf} = f_{be} = -0.707$ (Comp)

$f_{ec} = \dfrac{-f_{be}}{\cos 45} = \dfrac{0.707}{0.707} = +1.0$ (Tens)

$f_{ef} = f_{be} = -0.707$ (Comp) by inspection.

Example Problem - Truss Internally Indeterminate

Next we compute the Displacement in bar bf as a result of the release provided in the primary Structure. Let this Displacement be denoted by "Δ_{bf}"

Using "Virtual/Unit Load Method" we have

$$1\cdot\Delta_{bf} = \sum_{j=1}^{m} \frac{F\cdot f\cdot L}{AE}$$,

F = Member Forces in Primary Structure

f = Member Forces in Primary Structure due to Unit Load.

Member	F (KN)	L (m)	f (KN)	FfL	f^2L	$f\cdot F_{bf}$	$F+f\cdot F_{bf}$
ab	-75.42	5.66	0	0	0	0	-75.4
bc	-53.33	4.0	-0.707	+150.82	2.0	+5.05	-48.3
cd	-37.71	5.66	0	0	0	0	-37.71
ae	-6.67	4.0	0	0	0	0	- 6.67
ef	+26.66	4.0	-0.707	-75.4	2.0	+5.05	+ 37.71
fd	+26.66	4.0	0	0	0	0	+26.67
be	+53.33	4.0	-0.707	-150.82	2.0	+5.05	+58.38
bf	0	5.66	+1.0	0	5.66	-7.14	- 7.14
ce	+37.71	5.66	+1.0	+213.44	5.66	-7.14	+30.57
cf	0	4.0	-0.707	0	2.0	+5.05	+ 5.05
Σ				+138.04	+19.32		

$$1\cdot\Delta_{bf} = \Delta_{bf} = \sum_{j=1}^{m} \frac{FfL}{AE} = \frac{138.04}{AE}$$

$$1\cdot\delta_{bf} = \delta_{bf} = \sum_{j=1}^{m} \frac{f^2L}{AE} = \frac{19.32}{AE}$$

$$\Delta_{bf} + F_{bf}\cdot\delta_{bf} = 0$$

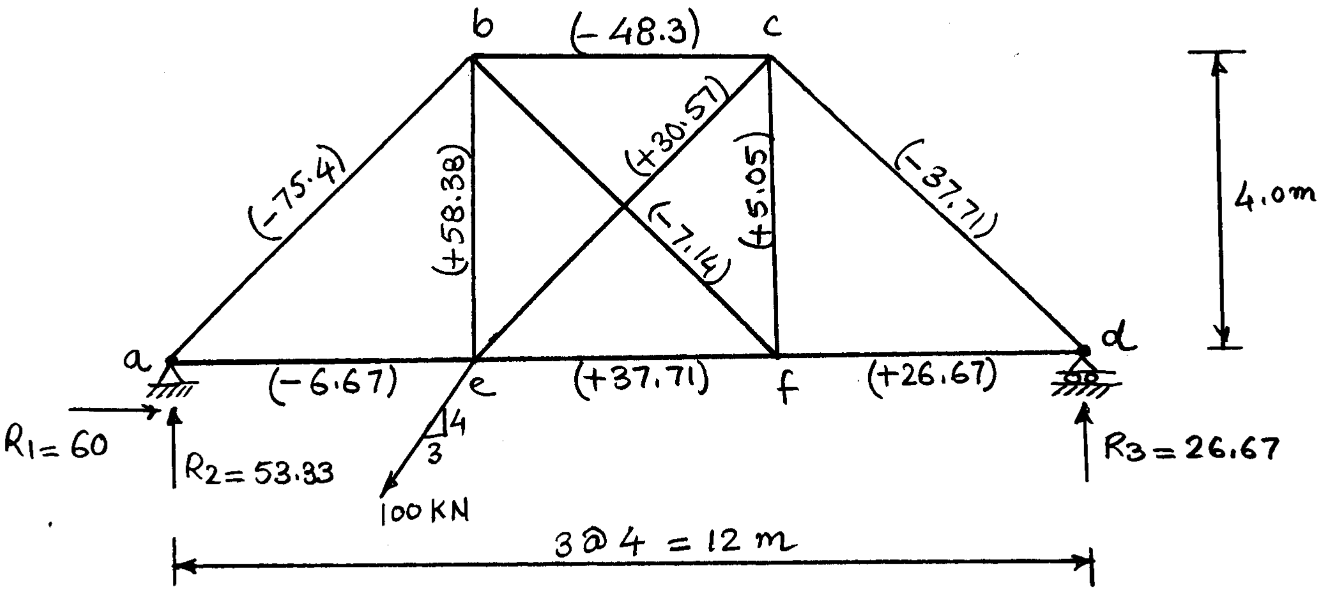
$$\Rightarrow F_{bf} = -\frac{\Delta_{bf}}{\delta_{bf}}$$

$$= -\frac{138.04}{AE} \times \frac{AE}{19.32}$$

$$\boxed{F_{bf} = -7.14 \text{ (Comp)}}$$

Compatibility Method of Analysis

Example Problem – Truss – Internally Indeterminate



FINAL MEMBER FORCES