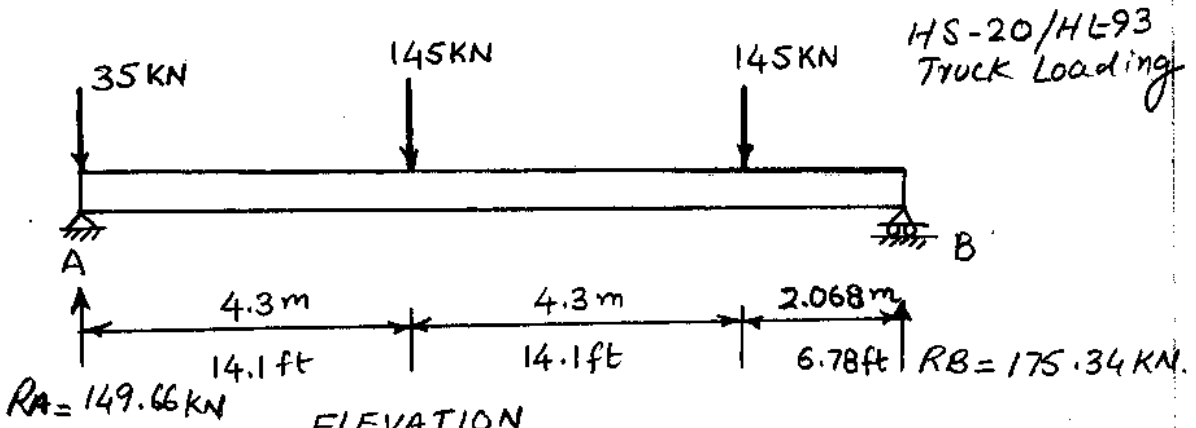
Example 6.1 Barker & Puckett

For the slab Girder Bridge shown below find the following:

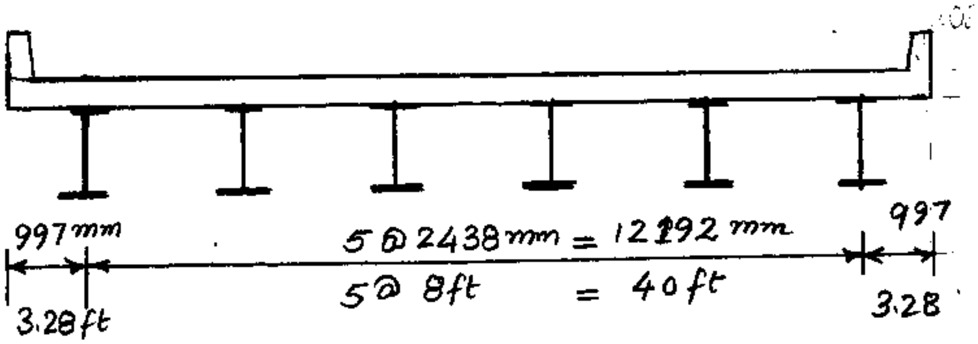
Support Reactions, shears and Bending Moments in the interior and exterior girders for the case of one-lane and two-lanes loaded.

Use Beam-Line / Distribution Factor Method

Use AASHTO DESIGN TRUCK



ELEVATION



X-SECTION

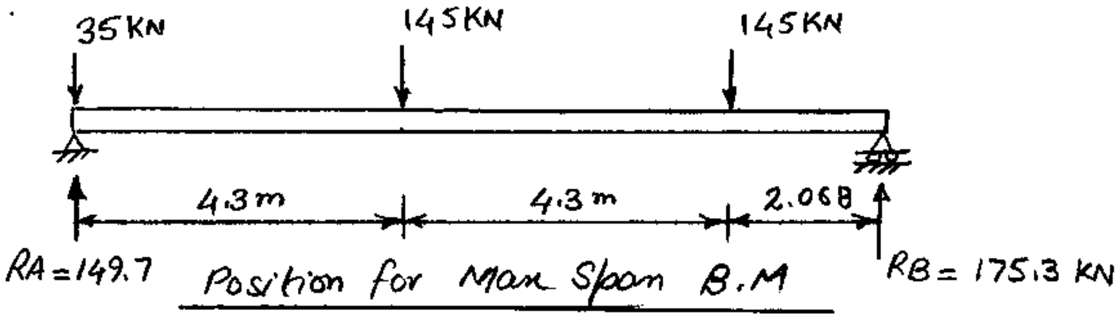
Bridge Reactions

$$R_B \times 10.668 = 145 \times 4.3 + 145 \times 8.6$$

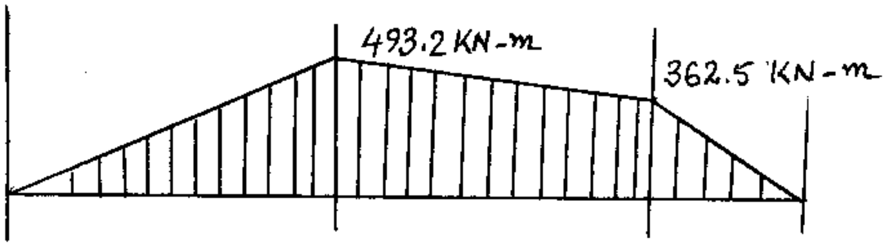
$$R_B = \frac{18705}{10.668} = 175.34 \text{ KN} \uparrow$$

$$R_A = 35 + 2 \times 145 - 175.34 = 149.66 \text{ KN} \uparrow$$

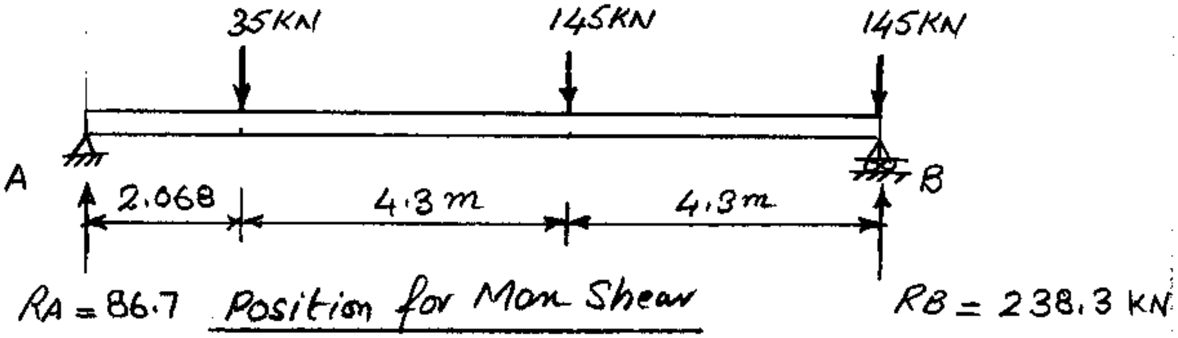
Example 6.1 Barker & Puckett (Contd.)



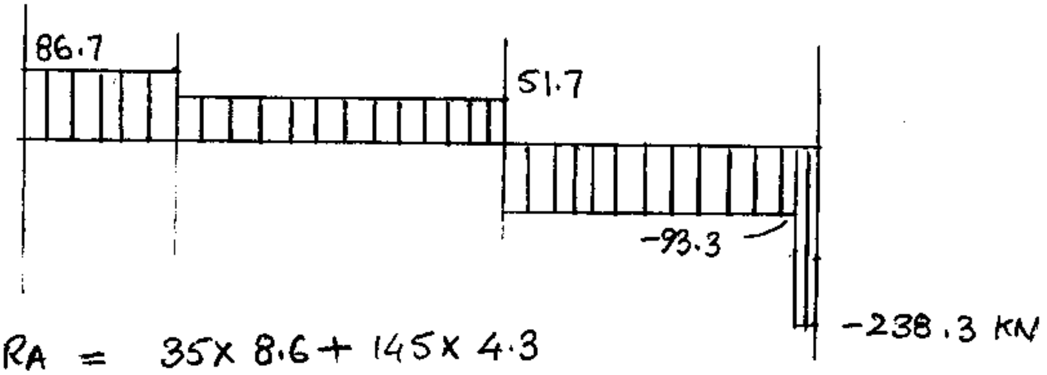
Position for Max Span B.M



Bending Moment



Position for Max Shear



$$10.668\, R_A = 35 \times 8.6 + 145 \times 4.3$$

$$R_A = \frac{924.5}{10.668} = 86.7 \text{ kN} \uparrow$$

$$R_B = 35 + 2 \times 145 - 86.7 = 238.3 \text{ kN} \uparrow$$

Example 6.1/6.2 Barker & Puckett (Contd.)

Determine AASHTO Distribution Factors for the Bridge

Girder Spacing = S = 8ft = 2438 mm

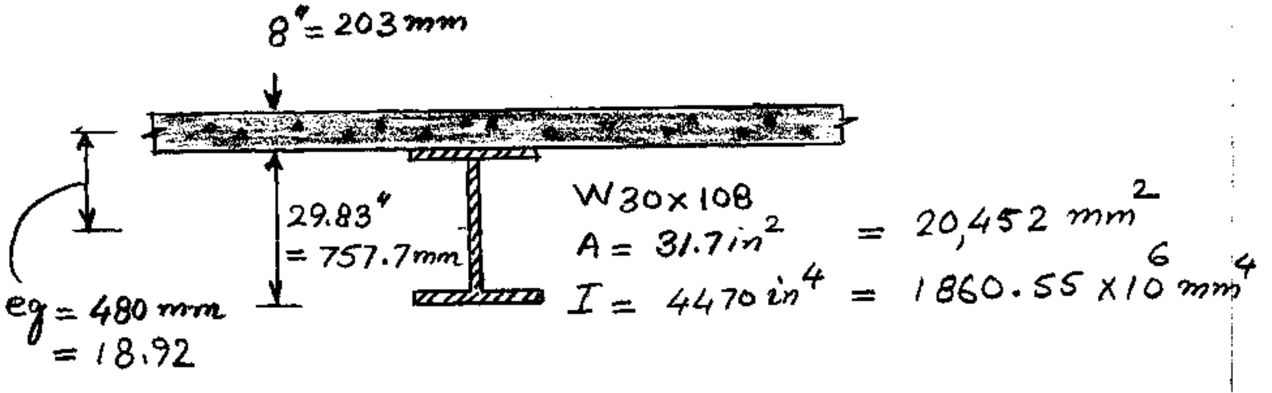
Span Length = L = 35ft = 10 668 mm

Deck thickness = t_s = 8 in. = 203 mm

Deck Modulus of Elasticity = E_c = 3600 ksi = 24,827.6 MPa

Girder Modulus of Elasticity = E_s = 29000 ksi = 200,000 MPa

$$\text{Modular Ratio} = n = \frac{E_s}{E_c} = \frac{200{,}000}{24{,}827.6} = 8.06 \simeq \underline{\underline{8.0}}$$



W30×108

$A = 31.7\,in^2 = 20{,}452\ mm^2$

$I = 4470\,in^4 = 1860.55 \times 10^6\ mm^4$

$$\text{Girder Eccentricity} = e_g = \frac{t_s}{2} + \frac{d}{2}$$

$$= \frac{29.83}{2} + \frac{8}{2} = 18.92\ in = 480\ mm$$

$$\text{Stiffness Parameter} = K_g = n\left(I_g + A_g e_g^2\right)$$

$$= 8\left[4470 + 31.7 \times (18.92)^2\right]$$

$$= 126{,}540.3\ in^4 = 52670 \times 10^6\ mm^4$$

Example 6.1/6.2 Barker & Puckett (Contd.)

AASHTO Distribution Factors

Refer AASHTO LRFD Table 4-6.2.2b-1

mg^{SI}_{moment} = Distribution Factor for moment in interior girder for single Lane Loaded.

$$= 0.06 + \left(\frac{S}{4300}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{L\,t_s^3}\right)^{0.1}$$

$$= 0.06 + \left(\frac{2438}{4300}\right)^{0.4}\left(\frac{2438}{10668}\right)^{0.3}\left(\frac{52670\times10^6}{10668\times(203)^3}\right)^{0.1}$$

$$= 0.06 + (0.5669)^{0.4}(0.2285)^{0.3}(0.5902)^{0.1}$$

$$= 0.545$$

mg^{MI}_{moment} = Distribution Factor for moment in Interior Girder for multiple Lanes Loaded

$$= 0.075 + \left(\frac{S}{2900}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{L\,t_s^3}\right)^{0.1}$$

$$= 0.075 + \left(\frac{2438}{2900}\right)^{0.6}\left(\frac{2438}{10668}\right)^{0.2}\left(\frac{52670\times10^6}{10668\times(203)^3}\right)^{0.1}$$

$$= 0.711$$

Example 6.1/6.2 Barker & Puckett

mg^{ME}_{moment} = Distribution Factor for moment in exterior girder for multiple lanes loaded.

$$mg^{ME}_{moment} = e \cdot mg^{MI}_{moment}$$

where,

e = Adjustment Factor

mg^{MI}_{moment} = Distribution Factor for moment in interior girder for multiple Lanes Loaded.

$$e = 0.77 + \frac{de}{2800\,mm} \geqslant 1.0$$ — Ref: Table 4-6.2.2d1

de = Distance from the exterior beam to the interior edge of the curb or traffic barriar (mm)

$$de = 3ft - 3\tfrac{1}{4}\,in - 1\,ft\ 3\tfrac{1}{4}''$$

$$= 2\,ft \qquad = 610\,mm$$

$$e = 0.77 + \frac{610}{2800} \geqslant 1.0 \qquad = 0.98 \geqslant 1.0$$

$$\Rightarrow e = 1.0$$

$$mg^{ME}_{moment} = e \cdot mg^{MI}_{m}$$

$$= 1.0 \times 0.711$$

$$mg^{ME}_{moment} = 0.711$$

Enample 6.1/6.2 Barker & Puckett

Find Distribution Factor For shear

mg^{SI}_{Shear} = Distribution Factor for Interior Girder for shear for single Lane Loaded.

$= 0.36 + \frac{S}{7600}$ mm

Ref: Table 4-6.2.2.3a1
AASHTO LRFD

$= 0.36 + \frac{2438}{7600}$

$mg^{SI}_{Shear} = 0.68$

$$mg^{MI}_{Shear} = 0.2 + \frac{S}{3600} - \left(\frac{S}{10700}\right)^{2.0}$$

$$mg^{MI}_{Shear} = 0.2 + \frac{2438}{3600} - \left(\frac{2438}{10700}\right)^{2.0} = 0.83$$

mg^{ME}_{Shear} = Distribution Factor for Exterior Girder for shear for multiple lanes loaded.

$= e \cdot mg^{MI}_{Shear}$

Ref: Table 4.6.2.2.3b-1
AASHTO LRFD

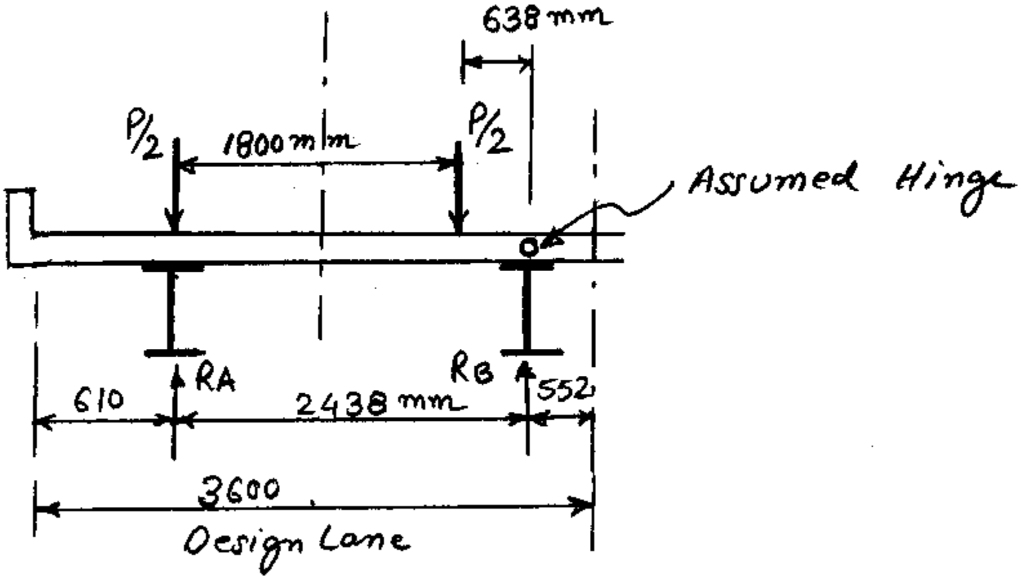
$$e = 0.6 + \frac{ds}{3000} = 0.6 + \frac{610}{3000} = 0.80$$

$$mg^{ME}_{Shear} = 0.80 \times 0.83 = 0.664$$

Example 6.1/6.2, Barker & Puckett

mg^{SE}_{shear} = Distribution Factor for exterior girder for shear for single Lane Loaded.
(To be determined by Lever Rule)

Ref: Table 4.6.2.2.3-b-1
AASHTO LRFD.



By Lever Rule method we have

$$2438 \times RA = \frac{P}{2} \times 638 + \frac{P}{2} \times 2438$$

$$= P\left(\frac{638 + 2438}{2}\right) = 1538\,P$$

$$RA = \frac{1538}{2438}\,P = 0.631\,P$$

$$RB = (1 - 0.631)\,P = 0.369\,P$$

$$\Rightarrow mg^{SE}_{shear} = 0.631 \times \underbrace{1.2}_{\text{Multiple Presence Factor Table 3-6.1.1.2.1 AASHTO}}$$

$$mg^{SE}_{shear} = \underline{0.757}$$

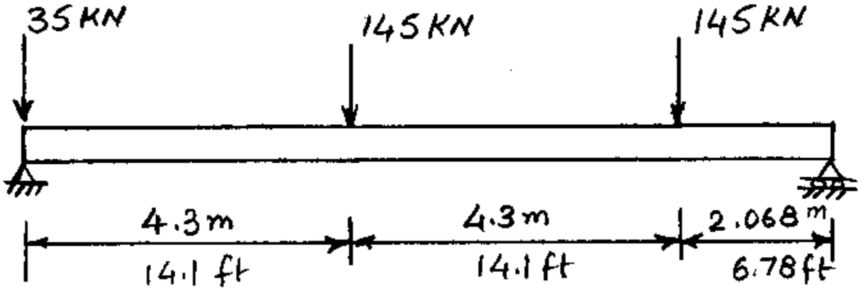
Example 6.1/6.2 Barker & Puckett

AASHTO DISTRIBUTION FACTOR RESULTS

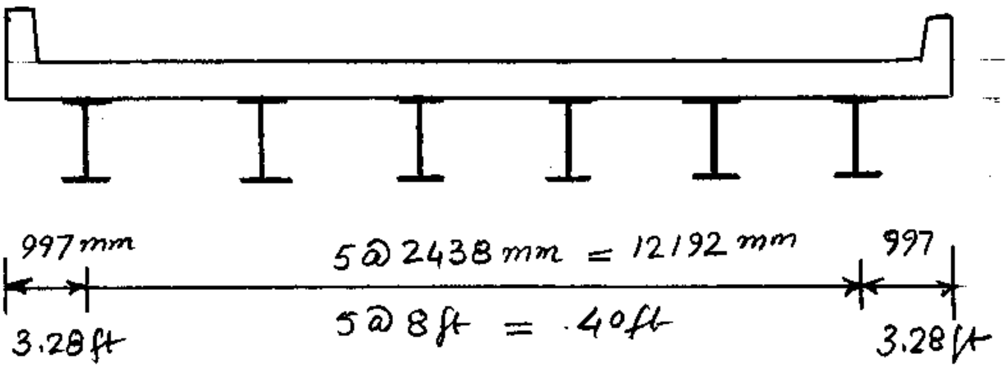
Girder Location	No. of Lanes Loaded	Moment (KN-m)	Moment Dist. Factor	Girder Moment (KN-m)	Simple Beam Shear (KN)	Shear Dist. Factor	Girder Shear (KN)
Exterior	1	498.7	0.75	374.0	238.3	0.75	178.7
Exterior	2		0.71	354.1		0.66	157.3
Interior	1		0.54	269.3		0.68	162.0
Interior	2		0.711	354.1		0.83	195.4

Example 6.3 Barker & Puckett

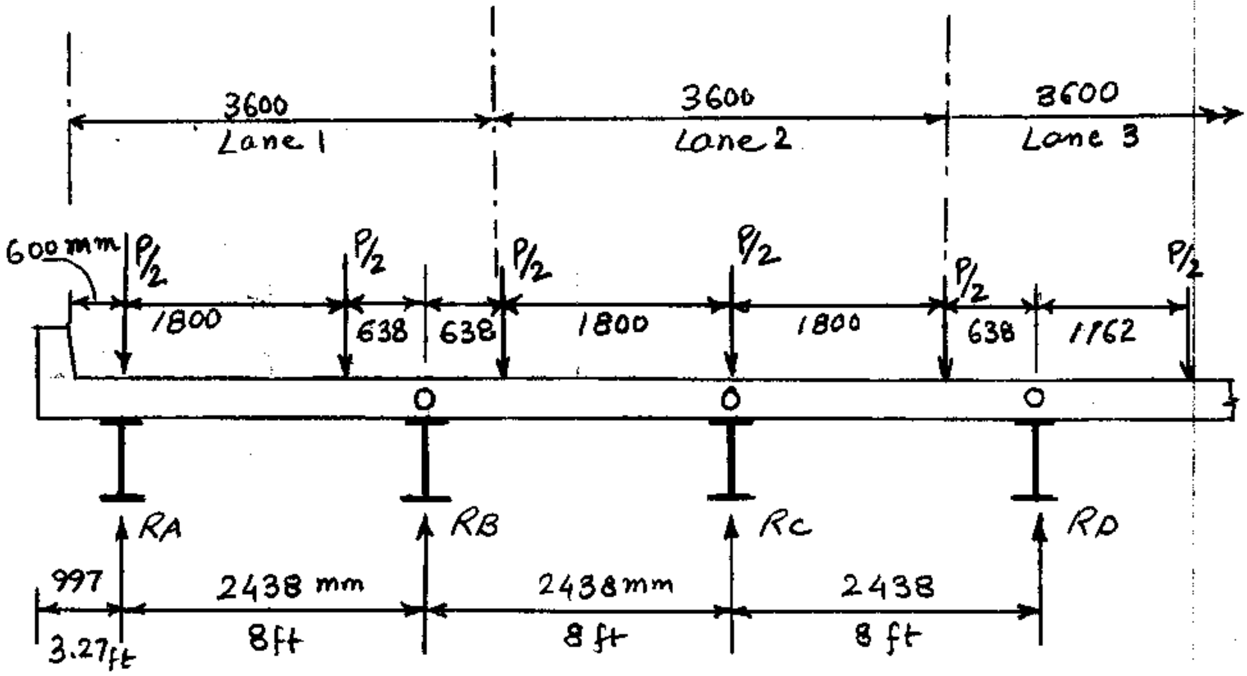
Use the "Lever Rule Method" to determine Distribution Factors for the Bridge shown below, that was solved in Example 6.1 & 6.2 using AASHTO Distribution Factors



ELEVATION



Example 6.3 Barker & Puckett



The Bridge Deck is divided into Design Lanes, each 3600 mm wide and the Design Truck is placed In each Lane as shown to maximize the effect on girders as shown. Attempt is made that truck from adjacent lane does not encroach onto the the neighboring Lane

Hinges are assumed to form on top of each girder which simplifies the analysis and computations considerably.

Taking moments at B

$$R_A \times 2438 = \left(\frac{P}{2}\right) \times 2438 + \left(\frac{P}{2}\right) \times 638$$

$$\Rightarrow R_A = \frac{2438+638}{2 \times 2438} P = 0.631\,P$$

$$\Rightarrow mg^{SE}_{Shear/Moment} = \underbrace{1.2}_{\text{Multiple Presence Factor}} \times 0.631 = 0.757$$

Example 6.3 Barker & Puckett

R_B considering exterior span to be loaded only

$$R_B = P - R_A = P - 0.631P = 0.369P$$

R_B Considering one interior Lane Loaded

$$2438\,R_B = \left(\tfrac{P}{2}\right) \times 1800$$

$$R_B = \frac{1800}{2 \times 2438}P = 0.369P$$

R_C Considering only Lane 2 Loaded

$$2438\,R_C = \tfrac{P}{2} \times 2438 + \tfrac{P}{2} \times 638$$

$$R_C = \frac{2438 + 638}{2 \times 2438}P = \underline{0.631P}$$

As $R_C > R_B$

we Adopt and Proceed with $R_C = 0.631P$

$\Rightarrow mg^{SI}_{Shear/Moment}$ = Distribution Factor for Interior Girder Single Lane Loaded

$$= \underbrace{(1.2)}_{\text{Multiple presence Factor}} \times 0.631 = 0.757$$

Example 6.3 Barker & Puckett

R_A remains unaffected whether single lane is loaded or 2 adjacent Lanes are loaded

hence

$$mg^{ME}_{shear/Moment} = \text{Distribution Factor for Exterior Girder Multiple Lanes Loaded}$$

$$= \underbrace{(1.0)}_{\text{Multiple Presence Factor}} \times 0.631 = \underline{\underline{0.631}}$$

$$mg^{MI}_{shear/Moment} = \text{Distribution Factor for Interior Girder Multiple Lanes Loaded} = ?$$

$$R_B - 2 \text{ Lanes Loaded} = R_B\Big|_{Lane 1} + R_B\Big|_{Lane 2}$$

$$= 0.369P + 0.369P$$

$$= 0.738P$$

$$R_C - 2 \text{ Lanes Loaded} = R_C\Big|_{Lane 2} + R_C\Big|_{Lane 3}$$

$$= \left\{\frac{P}{2} + \frac{P}{2} \times \frac{638}{2438}\right\} + \frac{P}{2} \times \frac{638}{2438}$$

$$= 0.631P + 0.262P$$

$$= \underline{0.893P} > R_B\,(0.738P)$$

Governs

$$\Rightarrow mg^{MI}_{shear/Moment} = \underbrace{1.0}_{\text{Multiple Presence Factor}} \times 0.893 = \underline{\underline{0.893}}$$

Example 6.3 Barker& Puckett

DISTRIBUTION FACTORS AND FORCES CALCULATED USING LEVER RULE

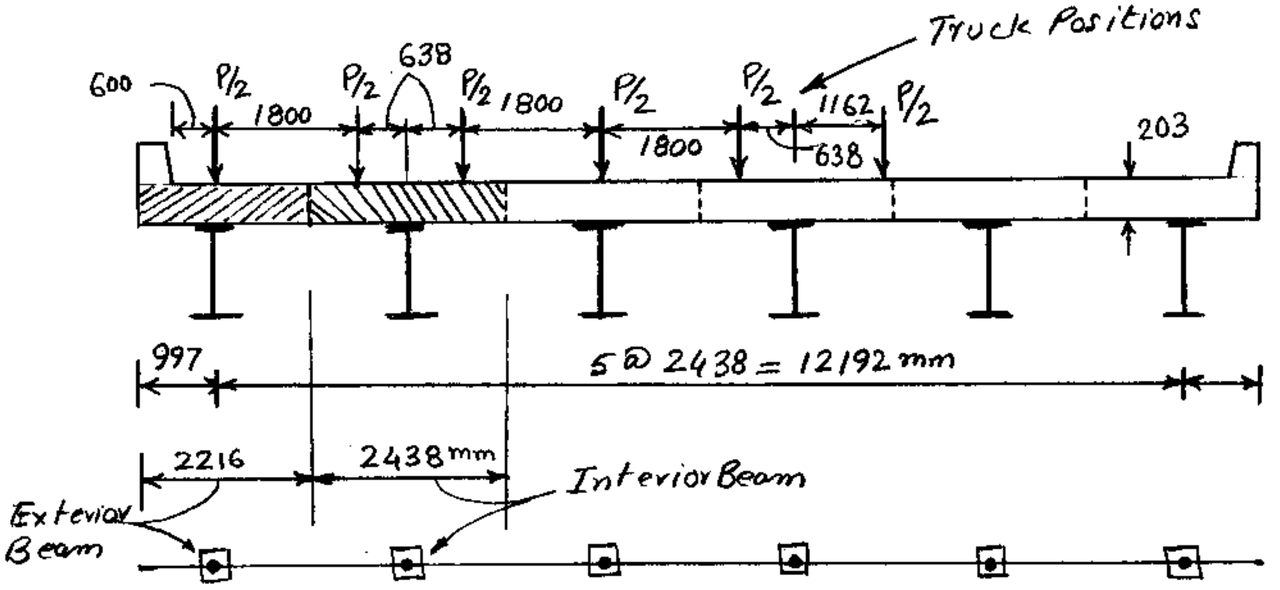
Girder Location	No. of Lanes Loaded	Moment (KN-m)	Moment/ Shear Dist. Factor	Girder Moment (KN-m)	Shear (KN)	Girder Shear (KN)
Exterior	1	498.7	0.757	377.5	238.3	180.4
Exterior	2		0.631	314.7		150.4
Interior	1		0.757	377.5		180.4
Interior	2		0.893	445.3		212.8

Example 6.4 Barker + Puckett

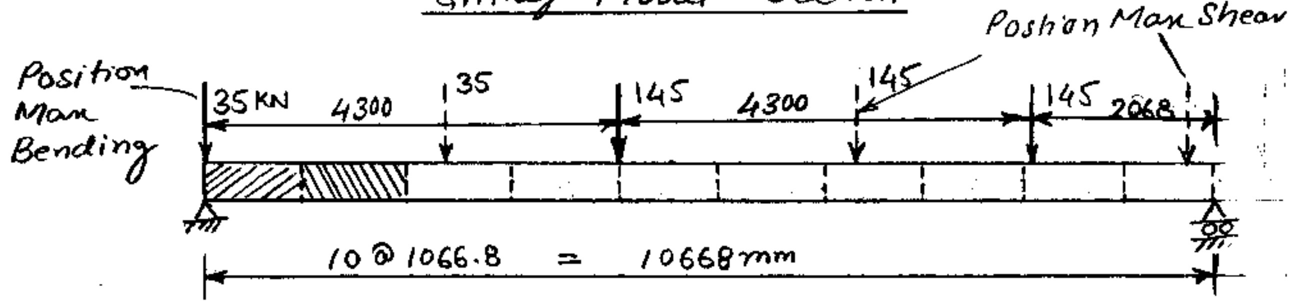
Grillage Analysis

1. Use the Grillage method of Analysis to determine maximum Bending Moments and Shears in the Bridge shown below and analyzed previously.

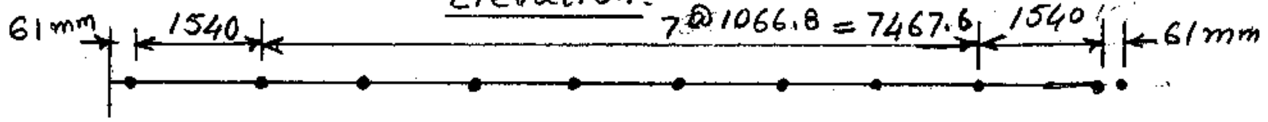
2. Utilize the information from the Grillage Analysis to determine the Distribution Factors for Moment and Shear for exterior and interior Girders for the Case of 1-lane and 2-lanes loaded.



Grillage Model – Section



Elevation.



Grillage Model Elevation

Example 6.4 Barker & Puckett

Grillage Analysis

Effective Flange width for interior beams is lesser of:

i) 1/4 of effective span $= \frac{1}{4} \times 10{,}668 = 2667$ mm

ii) 12 x Slab depth, plus greater of web thickness or 1/2 width of top flange

$\simeq 12 \times 203 \qquad = 2436$ mm

iii) c/c spacing of beams $= 2438$ mm

$\Rightarrow$ Adopt Effective Flange Width be $= \underline{\underline{2438}}$ mm

Governs

For Exterior Beam, Effective Flange width may be taken as:

1/2 Effective Flange Width of Interior Beam

+ Plus least of:

i) $\frac{1}{8}$ of effective span length

$\frac{1}{2} \times 2438 + \frac{1}{8} \times 10668 \qquad = 2553$ mm

ii) 6 times Avg slab depth

$\simeq \frac{1}{2} \times 2438 + 6 \times 203 \qquad = 2437$ mm

iii) Width of the overhang

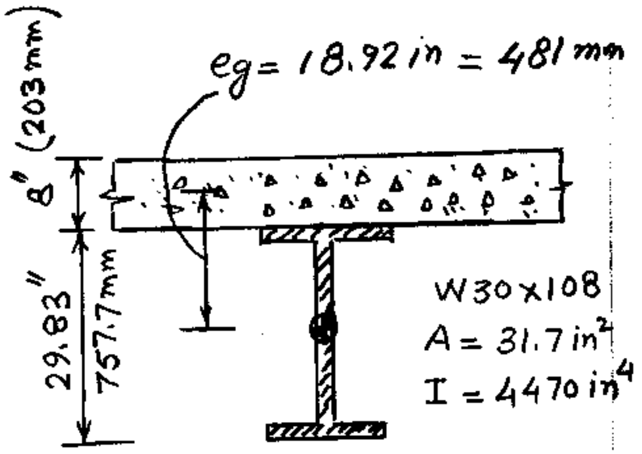
$\frac{1}{2} \times 2438 + 997 \qquad = \underline{\underline{2216}}$ mm

Governs

Example 6.4 Barker & Puckett

Section Properties

Girder Properties



$Es = 29000 \text{ ksi} = 200 \times 10^3 \text{ MPa}$

$Ag = 31.7 \text{ in}^2 = 20453 \text{ mm}^2$

$d = 29.83 \text{ in} = 757.7 \text{ mm}$

$eg = \text{Girder Eccentricity} = \frac{d}{2} + \frac{ts}{2}$

$= \frac{29.83}{2} + \frac{8}{2} \qquad = 18.92 \text{ in} = 481 \text{ mm}$

$Ig = 4470 \text{ in}^4$ (Non Composite Girder) $= 1860.6 \times 10^6 \text{ mm}^4$

Jg = Torsional Constant Non Composite Girder

$= 4.99 \text{ in}^4 \qquad = 2.077 \times 10^6 \text{ mm}^4$

Igt (Composite Girder) $= Ig + Ag\, eg^2$

$= 4470 + 31.7 \times (18.91)^2$

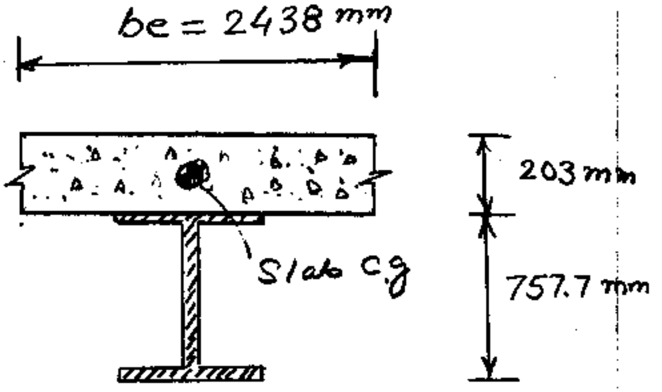
$= 15810 \text{ in}^2$ (steel) $= 6580.6 \times 10^6 \text{ mm}^4$

Example 6.4 Barker & Puckett

Grillage Element Properties

Longitudinal Element

Interior Girder



I_c = Moment of Inertia of Composite Girder

$= I_s + n\,I_{gt}$

I_s = Moment of Inertia of tributary portion of Slab

$= b_e \times i_s \quad = 2438 \times 700{,}000 \quad = 1706.6 \times 10^6 \text{ mm}^4$

I_{gt} = Moment of Inertia of Girder transformed to Slab centroid $= 6580.6 \times 10^6 \text{ mm}^4$

n = Modular Ratio

$= \dfrac{E_s}{E_c} = \dfrac{200 \times 10^3}{24830} = 8.05$

$\simeq \underline{\underline{8.0}}$ (say)

$I_c = 1706.6 \times 10^6 + 8 \times 6580.6 \times 10^6$

$= 54351 \times 10^6 \quad = 54{,}351 \times 10^6 \text{ mm}^4$

Example 6.4 Barker & Puckett

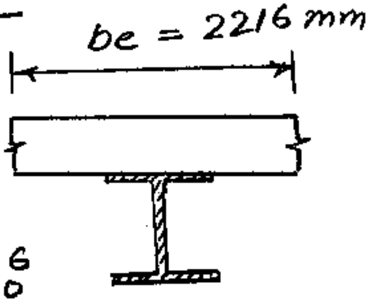
Grillage Longitudinal Element Props

J_c = Torsional Constant for Composite Girder

$= J_s + n J_g \quad = b_e j_s + n J_g$

$= 2438 \times 1400{,}000 + 8 \times 2.077 \times 10^6 \quad = 3430 \times 10^6\ mm^4$

Exterior Longitudinal Element



$I_c = b_e . i_s + n I_{gt}$

$= 2216 \times 700{,}000 + 8 \times 6580.6 \times 10^6$

$I_c = 54196 \times 10^6\ mm^4$

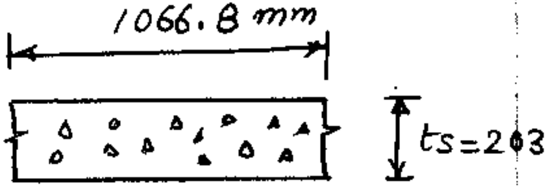
$J_c = b_e . j_s + n J_{gt}$

$= 2216 \times 1400{,}000 + 8 \times 2.077 \times 10^6$

$= 3119 \times 10^6\ mm^4$

Example 6.4 Barker & Puckett

Grillage Transverse Element Properties



$$I_c = be \cdot is$$

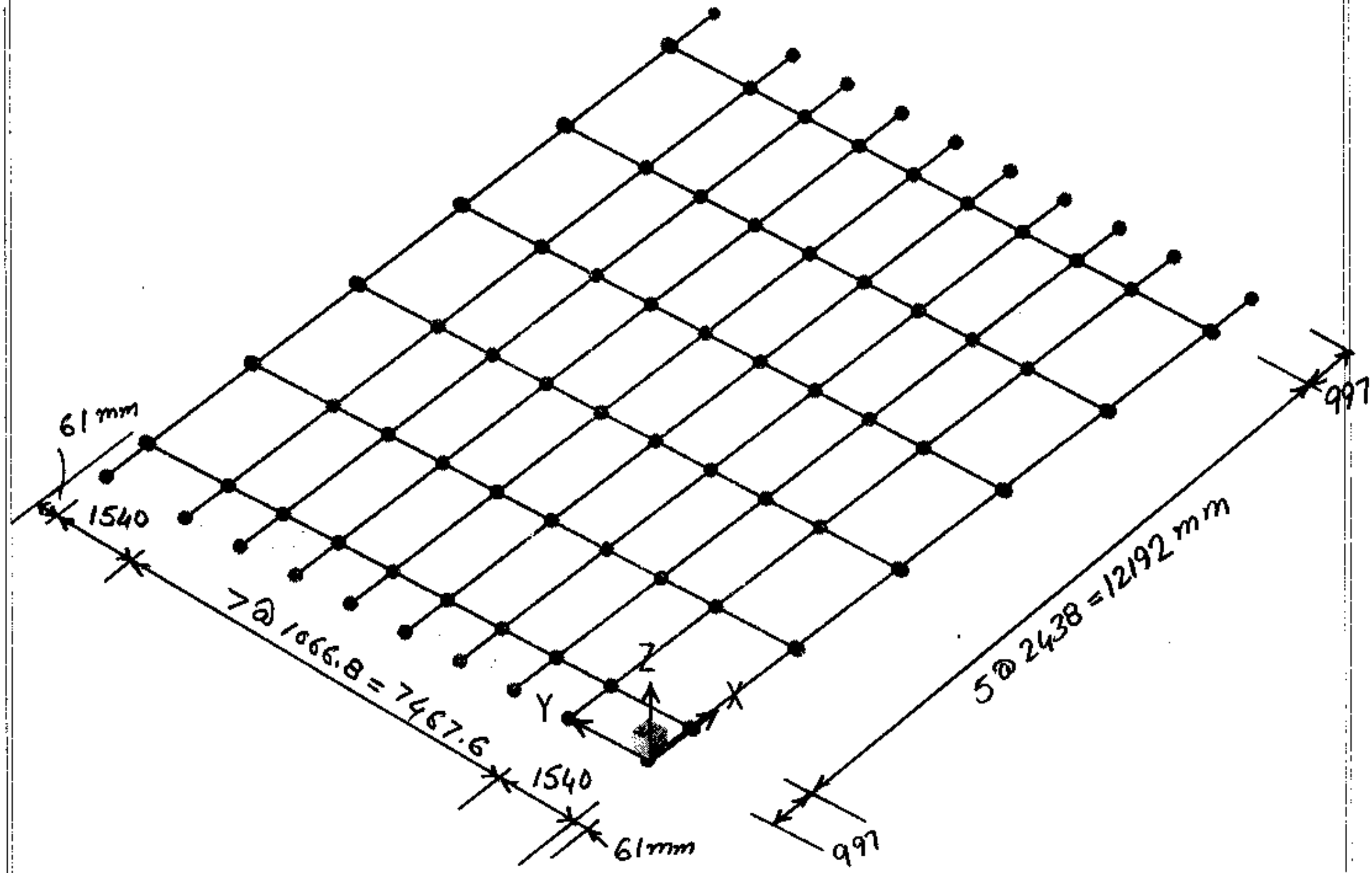
$$= 1066.8 \times 700{,}000$$

$$= 746.8 \times 10^{6} \text{ mm}^{4}$$

$$J_c = be \cdot js$$

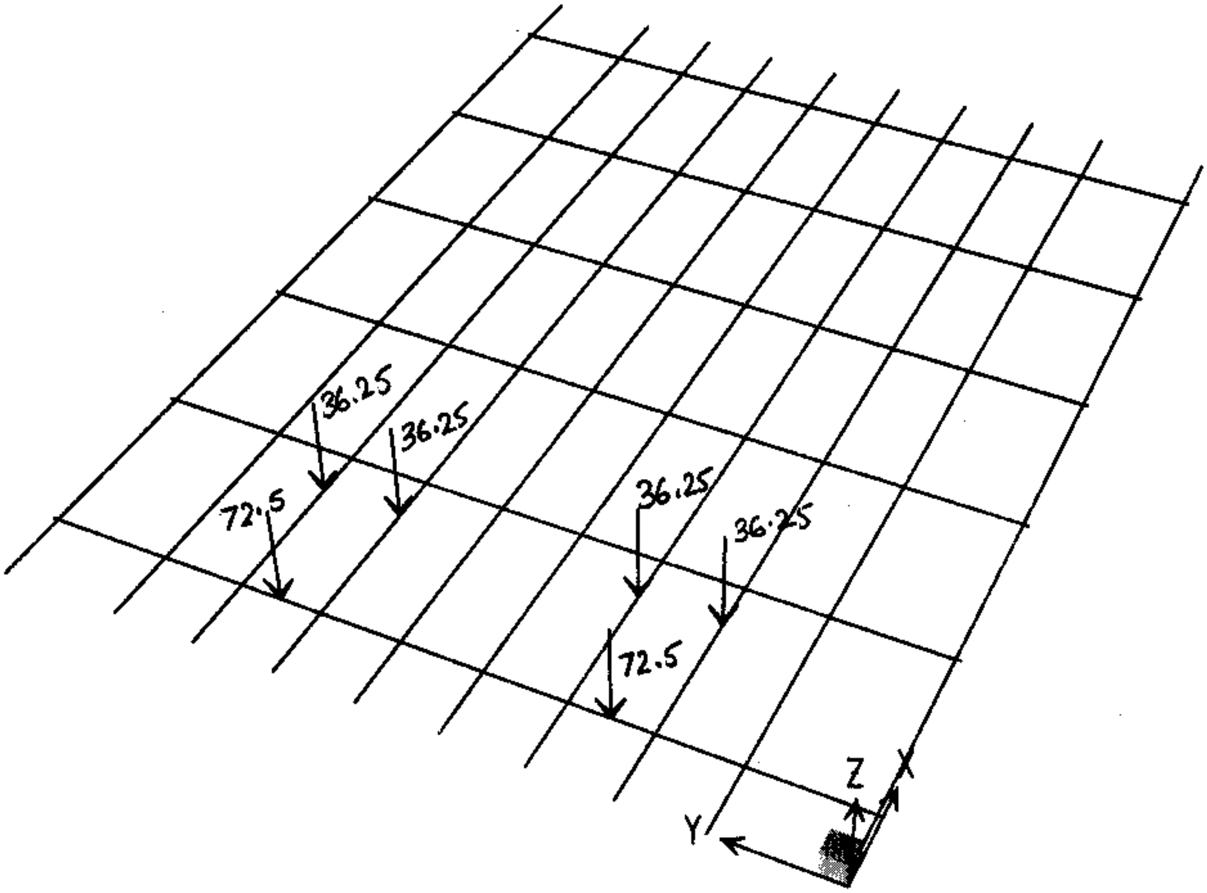
$$= 1066.8 \times 1400{,}000$$

$$= 1493.5 \times 10^{6} \text{ mm}^{4}$$

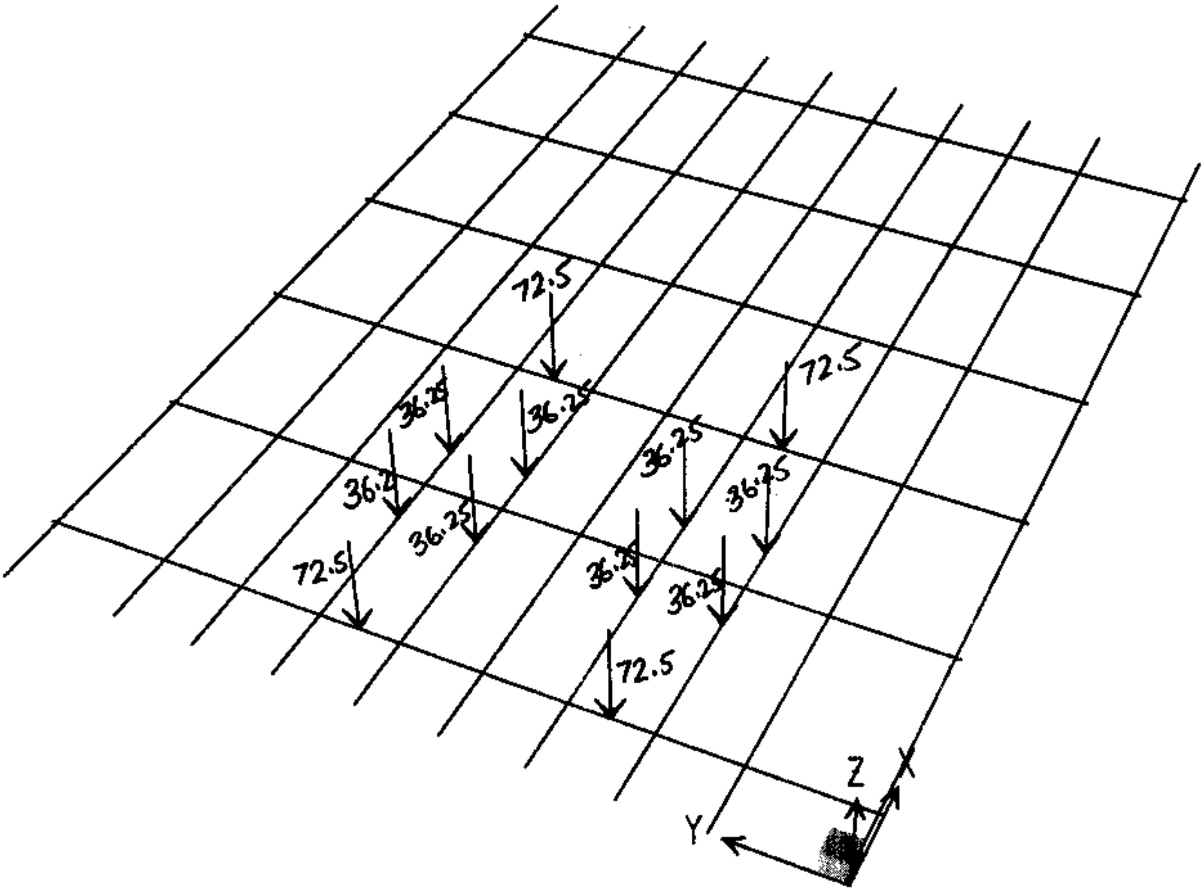


FINITE ELEMENT GRILLAGE MESH

22

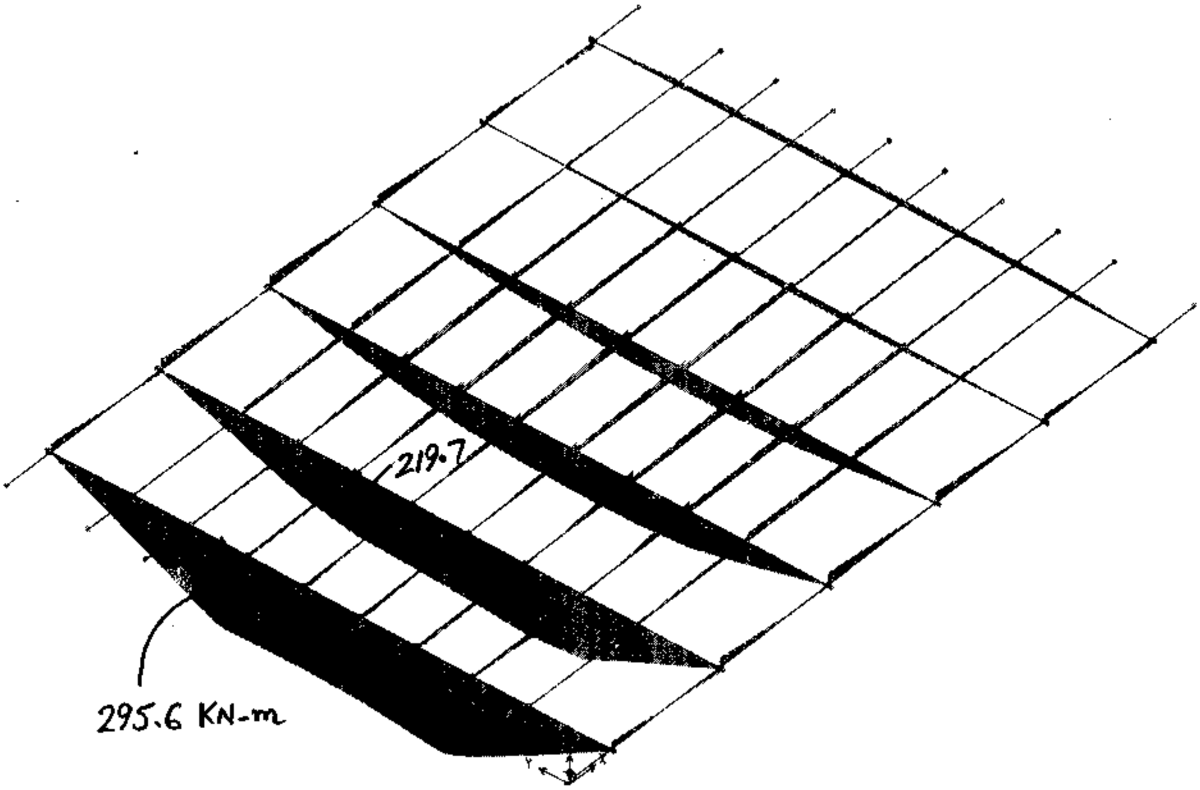


LOAD POSITIONS & VALUES
FOR SINGLE LANE LOADING.



LOAD POSITIONS & VALUES
FOR 2-LANES LOADED

BENDING MOMENTS / DISTRIBUTION FACTORS SINGLE LANE LOADING

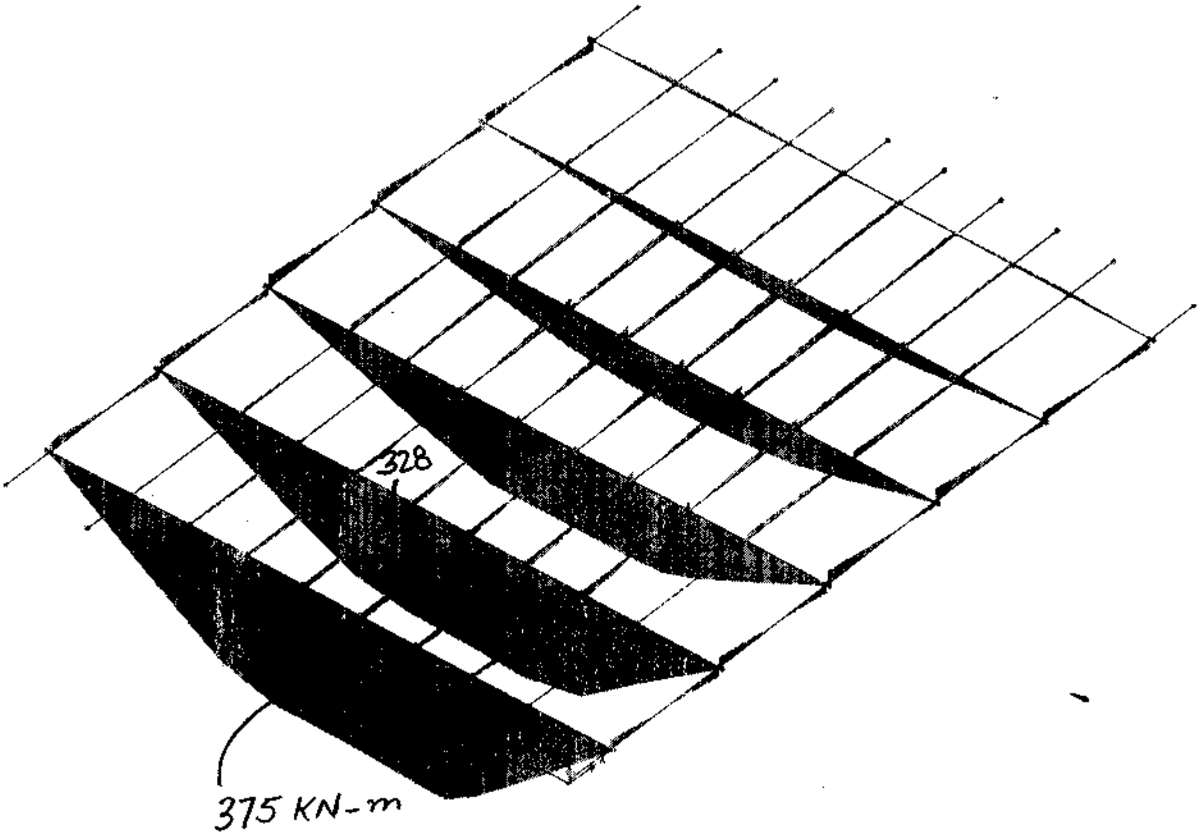


BEAM MOMENT = 498.7 KN-m

$mg^{SE}_{moment} = \frac{295.6}{498.7} = 0.59$

$mg^{SI}_{moment} = \frac{219.7}{498.7} = 0.44$

BENDING MOMENTS / DISTRIBUTION FACTORS
2_LANES LOADED.



BEAM MOMENT = 498.7 KN-m

$$mg^{mE}_{moment} = \frac{375}{498.7} = 0.75$$

$$mg^{mI}_{moment} = \frac{328}{498.7} = 0.66$$