

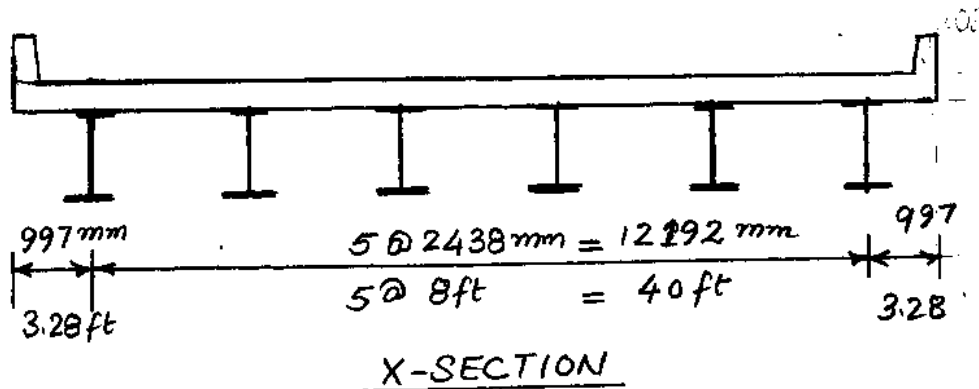
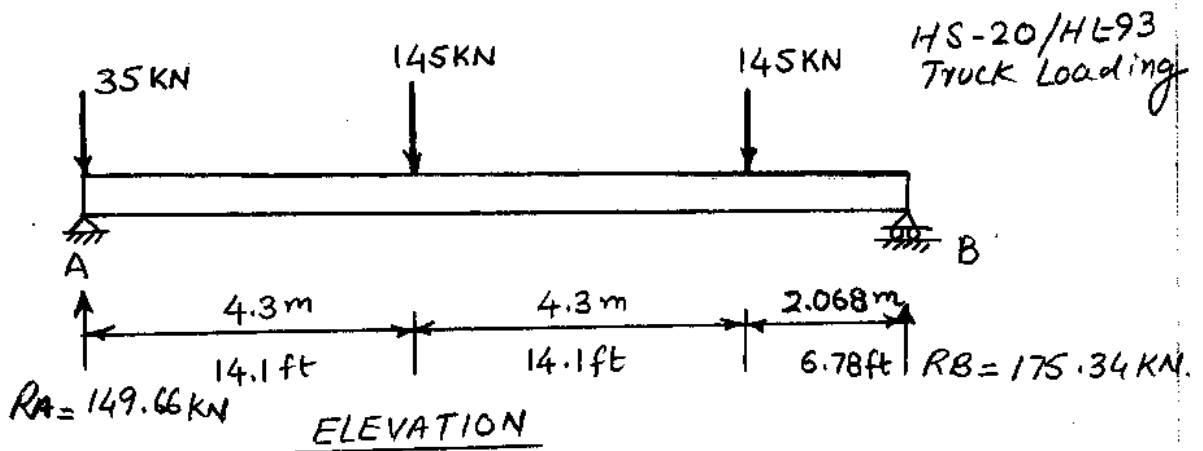
Example 6.1 Barker & Puckett

For the slab Girder Bridge shown below find the following:

Support Reactions, shears and Bending Moments in the interior and exterior girders for the case of one-lane and two-lanes loaded.

Use Beam-Line / Distribution Factor Method

Use AASHTO DESIGN TRUCK



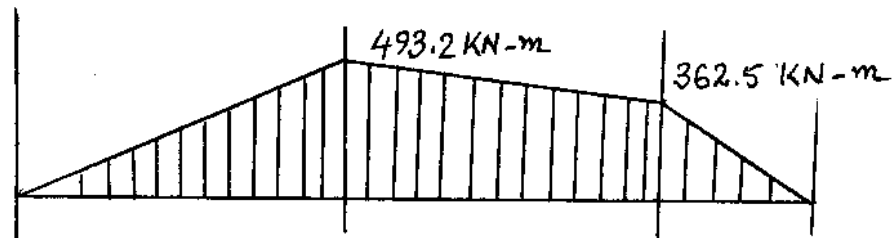
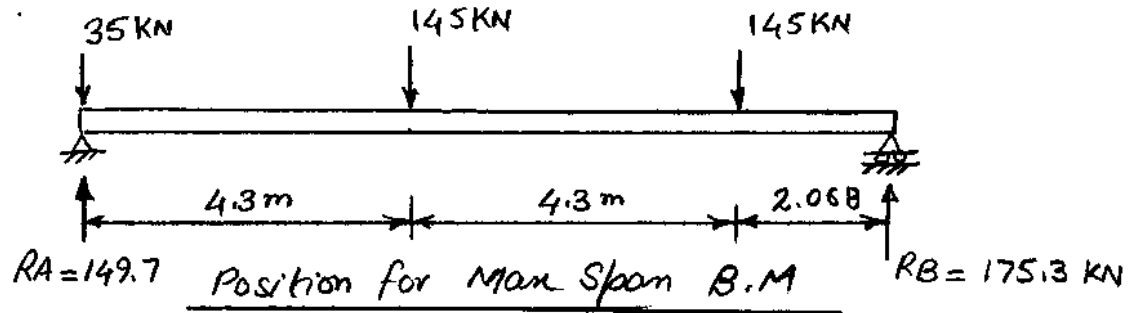
Bridge Reactions

$$R_B \times 10.668 = 145 \times 4.3 + 145 \times 8.6$$

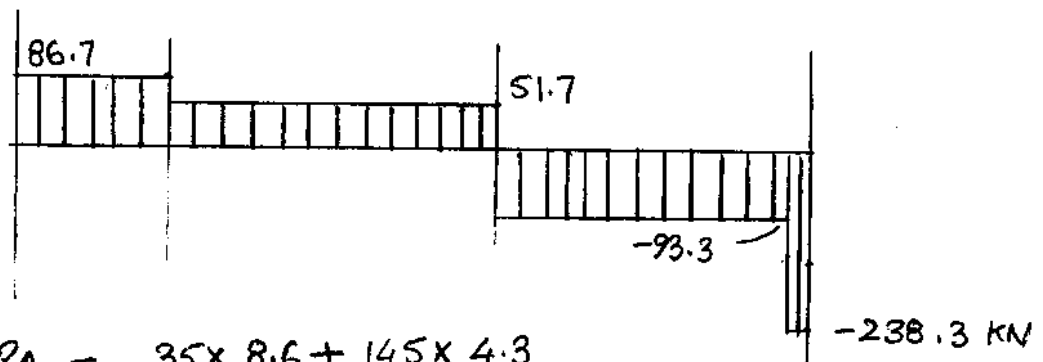
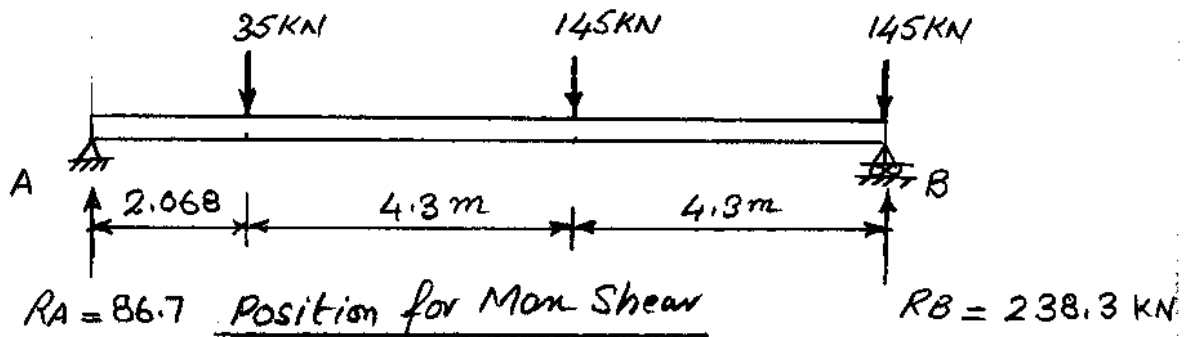
$$R_B = \frac{18705}{10.668} = 175.34 \text{ kN} \uparrow$$

$$R_A = 35 + 2 \times 145 - 175.34 = 149.66 \text{ kN} \uparrow$$

Example 6.1 Barker & Puckett (Contd.)



Bending Moment



$$10.668 R_A = 35 \times 8.6 + 145 \times 4.3$$

$$R_A = \frac{924.5}{10.668} = 86.7 \text{ kN} \uparrow$$

$$R_B = 35 + 2 \times 145 - 86.7 = 238.3 \text{ kN} \uparrow$$

Example 6.1/6.2 Barker & Puckett (Contd.)

Determine AASHTO Distribution Factors for the Bridge

$$\text{Girder Spacing} = S = 8 \text{ ft} = 2438 \text{ mm}$$

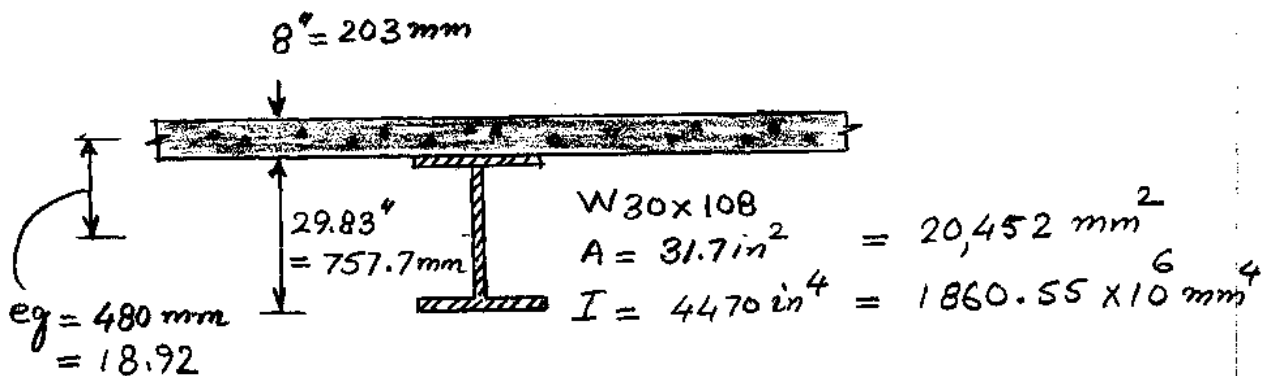
$$\text{Span Length} = L = 35 \text{ ft} = 10668 \text{ mm}$$

$$\text{Deck thickness} = t_s = 8 \text{ in.} = 203 \text{ mm}$$

$$\text{Deck Modulus of Elasticity} = E_c = 3600 \text{ ksi} = 24,827.6 \text{ MPa}$$

$$\text{Girder Modulus of Elasticity} = E_s = 29000 \text{ ksi} = 200,000 \text{ MPa}$$

$$\text{Modular Ratio} = n = \frac{E_s}{E_c} = \frac{200,000}{24,827.6} = 8.06 \approx \underline{\underline{8.0}}$$



$$\begin{aligned} \text{Girder Eccentricity} = e_g &= \frac{t_s}{2} + \frac{d}{2} \\ &= \frac{29.83}{2} + \frac{8}{2} = 18.92 \text{ in} = 480 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Stiffness Parameter} = K_g &= n (I_g + A_g e_g^2) \\ &= 8 [4470 + 31.7 \times (18.92)^2] \\ &= 126,540.3 \text{ in}^4 = 52670 \times 10^6 \text{ mm}^4 \end{aligned}$$

Example 6.1/6.2 Barker & Puckett (Contd.)

AASHTO Distribution Factors

Refer AASHTO LRFD Table 4-6.2.2b-1

$m_{\text{moment}}^{\text{SI}}$ = Distribution Factor for moment in interior girder for single Lane Loaded.

$$\begin{aligned}
 &= 0.06 + \left(\frac{S}{4300}\right)^{0.4} \left(\frac{S}{L}\right)^{0.3} \left(\frac{Kg}{L \text{ ts}^3}\right)^{0.1} \\
 &= 0.06 + \left(\frac{2438}{4300}\right)^{0.4} \left(\frac{2438}{10668}\right)^{0.3} \left(\frac{52670 \times 10^6}{10668 \times (203)^3}\right)^{0.1} \\
 &= 0.06 + (0.5669)^{0.4} (0.2285)^{0.3} (0.5902)^{0.1} \\
 &= 0.545
 \end{aligned}$$

$m_{\text{moment}}^{\text{MI}}$ = Distribution Factor for moment in Interior Girder for multiple Lanes Loaded

$$\begin{aligned}
 &= 0.075 + \left(\frac{S}{2900}\right)^{0.6} \left(\frac{S}{L}\right)^{0.2} \left(\frac{Kg}{L \text{ ts}^3}\right)^{0.1} \\
 &= 0.075 + \left(\frac{2438}{2900}\right)^{0.6} \left(\frac{2438}{10668}\right)^{0.2} \left(\frac{52670 \times 10^6}{10668 \times (203)^3}\right)^{0.1} \\
 &= 0.711
 \end{aligned}$$

Example 6.1/6.2 Barker & Puckett

$m_{\text{moment}}^{\text{ME}}$ = Distribution Factor for moment in exterior girder for multiple lanes loaded.

$$m_{\text{moment}}^{\text{ME}} = e \cdot m_{\text{moment}}^{\text{MI}}$$

where, e = Adjustment Factor

$m_{\text{moment}}^{\text{MI}}$ = Distribution Factor for moment in interior girder for multiple lanes loaded.

$$e = 0.77 + \frac{d_e}{2800 \text{ mm}} \geq 1.0 \quad \text{--- Ref: Table 4-6.2.2d.1}$$

d_e = Distance from the exterior beam to the interior edge of the curb or traffic barrier (mm)

$$\begin{aligned} d_e &= 3 \text{ ft} - 3 \frac{1}{4} \text{ in} - 1 \text{ ft} 3 \frac{1}{4} \text{ in} \\ &= 2 \text{ ft} &= 610 \text{ mm} \end{aligned}$$

$$e = 0.77 + \frac{610}{2800} \geq 1.0 \quad = 0.98 \geq 1.0$$

$$\Rightarrow e = 1.0$$

$$\begin{aligned} m_{\text{moment}}^{\text{ME}} &= e \cdot m_{\text{moment}}^{\text{MI}} \\ &= 1.0 \times 0.711 \end{aligned}$$

$$m_{\text{moment}}^{\text{ME}} = 0.711$$

Example G.1/6.2 Barker & Puckett

Find: Distribution Factor for shear

$m_{g_{Shear}}^{SI}$ = Distribution Factor for Interior Girder
for shear for single Lane Loaded.

$$= 0.36 + \frac{S}{7600 \text{ mm}}$$

Ref: Table 4-6.2.2.3a-1
AASHTO LRFD

$$= 0.36 + \frac{2438}{7600}$$

$$\underline{m_{g_{Shear}}^{SI} = 0.68}$$

$$m_{g_{Shear}}^{MI} = 0.2 + \frac{S}{3600} - \left(\frac{S}{10700} \right)^{2.0}$$

$$m_{g_{Shear}}^{MI} = 0.2 + \frac{2438}{3600} - \left(\frac{2438}{10700} \right)^{2.0} = \underline{0.83}$$

$m_{g_{Shear}}^{ME}$ = Distribution Factor for Exterior Girder
for shear for multiple lanes loaded.

$$= e \cdot m_{g_{Shear}}^{MI}$$

Ref: Table 4.6.2.2.3b-1
AASHTO LRFD

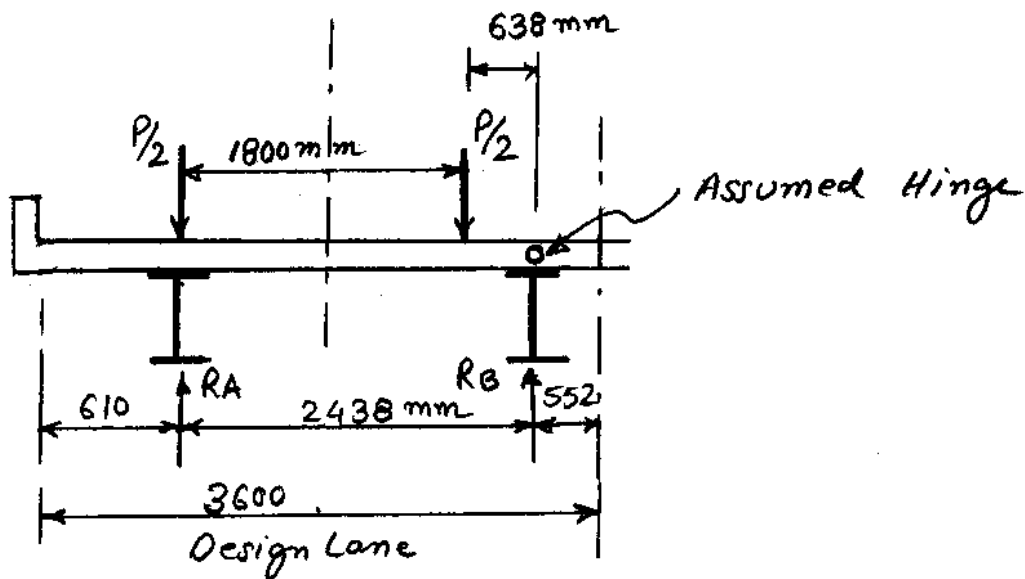
$$e = 0.6 + \frac{ds}{3000} = 0.6 + \frac{610}{3000} = 0.80$$

$$m_{g_{Shear}}^{ME} = 0.80 \times 0.83 = \underline{0.664}$$

Example 6.1/6.2, Barker & Puckett

m_{shear}^{SE} = Distribution Factor for exterior girder for shear for single Lane Loaded.
(To be determined by Lever Rule)

Ref: Table 4.6.2.2.3-b-1
AASHTO LRFD.



By Lever Rule method we have

$$2438 \times R_A = \frac{P}{2} \times 638 + \frac{P}{2} \times 2438$$

$$= P \left(\frac{638 + 2438}{2} \right) = 1538 P$$

$$R_A = \frac{1538}{2438} P = 0.631 P$$

$$R_B = (1 - 0.631) P = 0.369 P$$

$$\Rightarrow m_{shear}^{SE} = 0.631 \times 1.2$$

Multiple Presence
Factor Table 3-6.1.12.1 AASHTO

$$m_{shear}^{SE} = \underline{0.757}$$

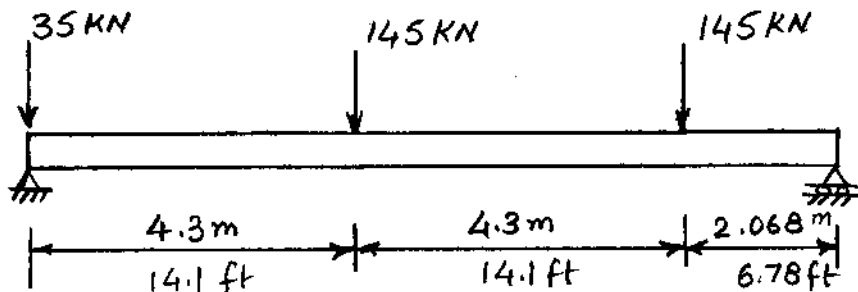
Example 6.1/6.2 Bankev & Puckett

AASHTO DISTRIBUTION FACTOR RESULTS

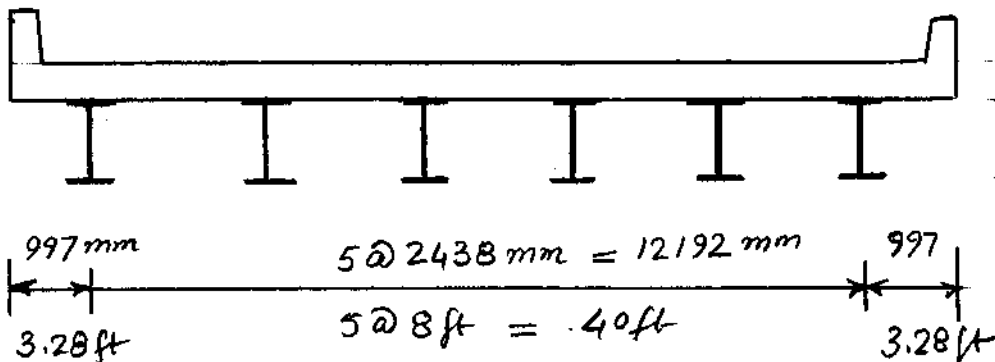
Girder Location	NO. of Lanes Loaded	Moment (KN-m)	Moment Dist. Factor	Girder Moment (KN-m)	Simple Beam Shear (KN)	Shear Dist. Factor	Girder Shear (KN)
Exterior	1	↑ 498.7	0.75	374.0	↑ 238.3	0.75	178.7
Exterior	2		0.71	354.1		0.66	157.3
Interior	1	↓	0.54	269.3	↓	0.68	162.0
Interior	2		0.711	354.1		0.83	195.4

Example 6.3 Barker & Puckett

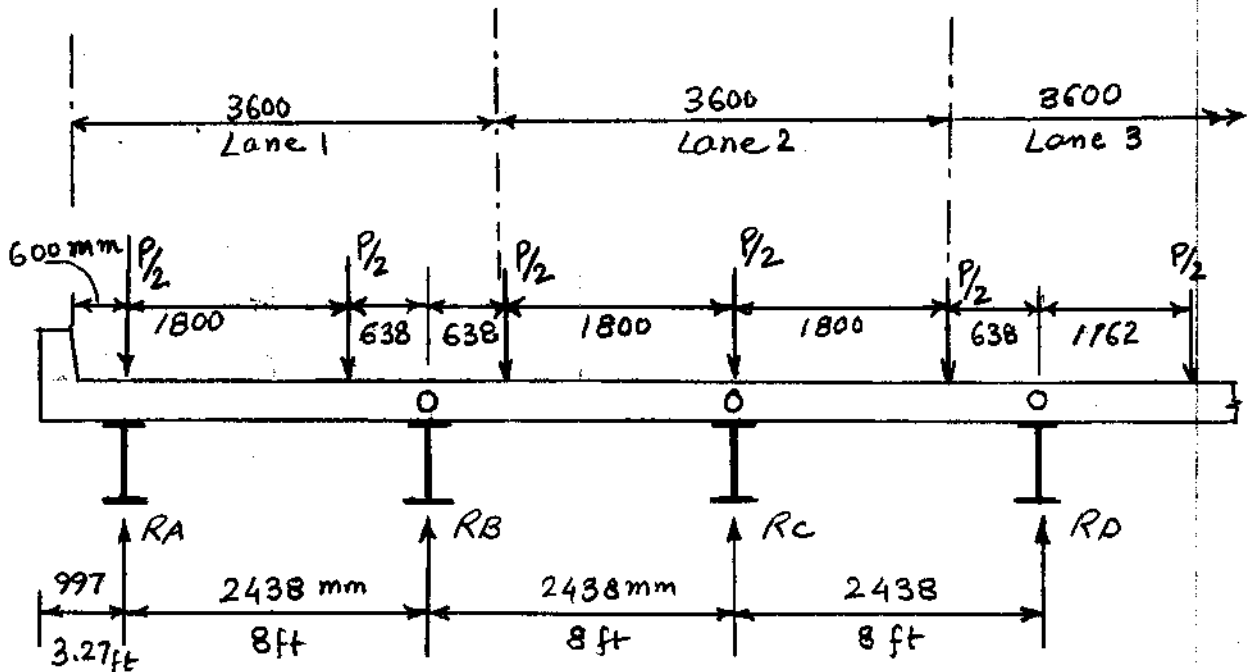
Use the "Lever Rule Method" to determine Distribution Factors for the Bridge shown below, that was solved in Example 6.1 & 6.2 using AASHTO Distribution Factors



ELEVATION



Example 6.3 Bunker & Puckett



The Bridge Deck is divided into Design Lanes, each 3600 mm wide and the Design Truck is placed in each Lane as shown to maximize the effect on girders as shown. Attempt is made that truck from adjacent lane does not encroach onto the neighboring lane.

Hinges are assumed to form on top of each girder which simplifies the analysis and computations considerably.

Taking moments at B

$$R_A \times 2438 = \left(\frac{P}{2}\right) \times 2438 + \left(\frac{P}{2}\right) \times 638$$

$$\Rightarrow R_A = \frac{2438 + 638}{2 \times 2438} P = 0.631 P$$

$$\Rightarrow m_g^{SE} \text{ Shear/Moment} = \underbrace{1.2 \times 0.631}_{\text{Multiple Presence Factor}} = 0.757$$

11

Example 6.3 Barker & Puckett

R_B considering exterior span to be loaded only

$$R_B = P - R_A = P - 0.631P = 0.369P$$

R_B considering one interior lane loaded

$$2438 R_B = \left(\frac{P}{2}\right) \times 1800$$

$$R_B = \frac{1800}{2 \times 2438} P = 0.369P$$

R_C considering only Lane 2 loaded

$$2438 R_C = \frac{P}{2} \times 2438 + \frac{P}{2} \times 638$$

$$R_C = \frac{2438 + 638}{2 \times 2438} P = \frac{0.631P}{1}$$

As $R_C > R_B$

we adopt and proceed with $R_C = 0.631P$

$$\begin{aligned} \Rightarrow m_g^{SI} \text{ Shear/Moment} &= \text{Distribution Factor for Interior Girder Single Lane Loaded} \\ &= \underbrace{(1.2)}_{\text{Multiple presence Factor}} \times 0.631 = 0.757 \end{aligned}$$

Example 6.3 Barker & Puckett

R_A remains unaffected whether single lane is loaded or 2 adjacent lanes are loaded

hence

$$m_g^{ME} \text{ Shear/Moment} = \text{Distribution Factor for Exterior Girder Multiple Lanes Loaded}$$

$$= \underbrace{(1.0)}_{\text{Multiple Presence Factor}} \times 0.631 = \underline{\underline{0.631}}$$

$$m_g^{MI} \text{ Shear/Moment} = \text{Distribution Factor for Interior Girder Multiple Lanes Loaded} = ?$$

$$R_B - 2 \text{ Lanes Loaded} = R_B|_{\text{Lane 1}} + R_B|_{\text{Lane 2}}$$

$$= 0.369P + 0.369P$$

$$= 0.738P$$

$$R_C - 2 \text{ Lanes Loaded} = R_C|_{\text{Lane 2}} + R_C|_{\text{Lane 3}}$$

$$= \left\{ \frac{P}{2} + \frac{P}{2} \times \frac{638}{2438} \right\} + \frac{P}{2} \times \frac{638}{2438}$$

$$= 0.631P + 0.262P$$

$$= \underline{\underline{0.893P}} > R_B (0.738P) \text{ Governs}$$

$$\Rightarrow m_g^{MI} \text{ Shear/Moment}$$

$$= \underbrace{1.0}_{\text{Multiple Presence Factor}} \times 0.893 = \underline{\underline{0.893}}$$

Example 6.3 Barker & Puckett

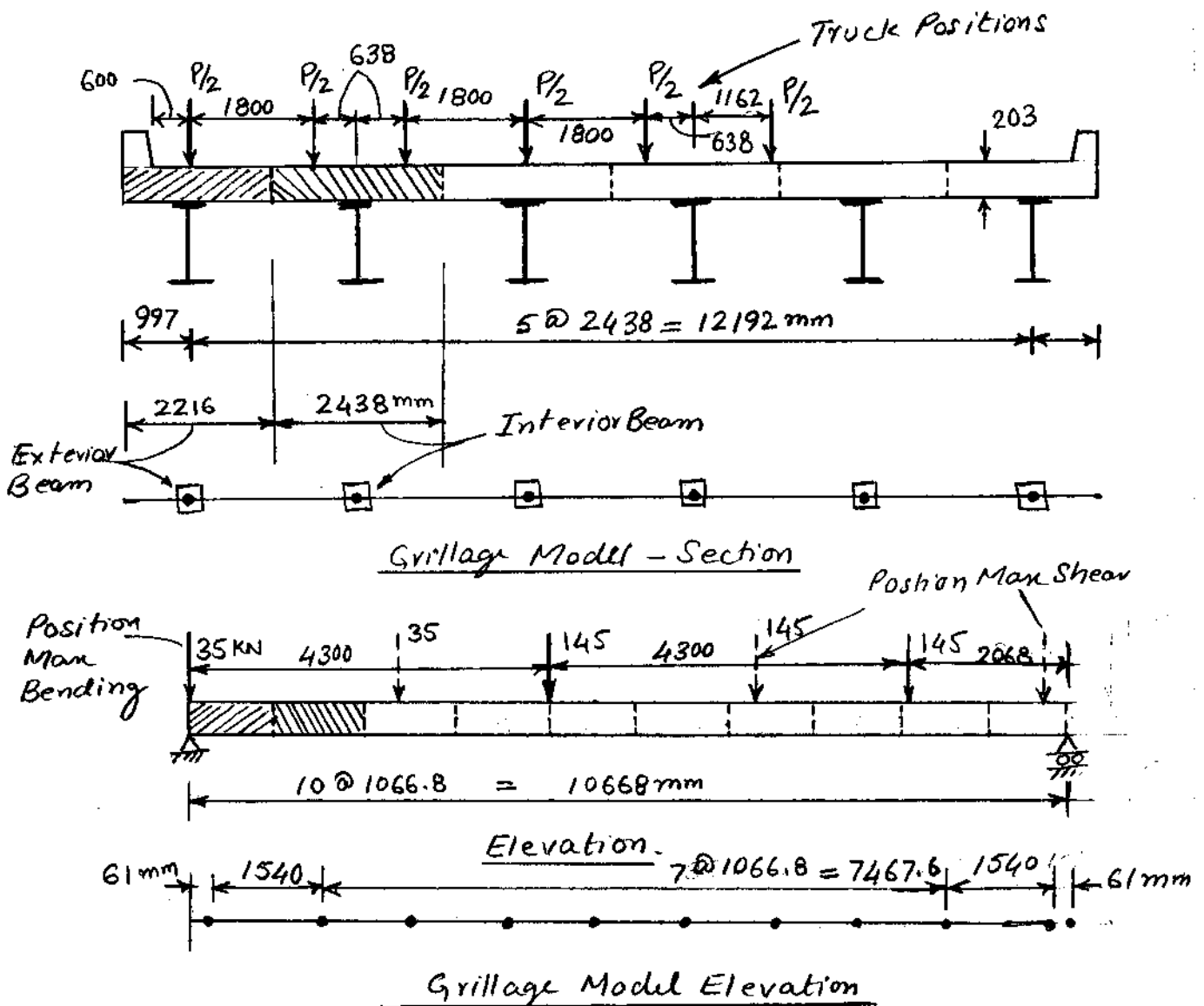
DISTRIBUTION FACTORS AND FORCES
CALCULATED USING LEVER RULE

Girder Location	No. of Lanes Loaded	Moment (KN-m)	Moment/Shear Dist. Factor	Girder Moment (KN-m)	Shear (KN)	Girder Shear (KN)
Exterior	1	↑ 498.7	0.757	377.5	↑ 238.3	180.4
Exterior	2		0.631	314.7		150.4
Interior	1	↓	0.757	377.5	↓	180.4
Interior	2		0.893	445.3		212.8

Example 6.4 Barker & Puckett

Grillage Analysis

1. Use the Grillage method of Analysis to determine maximum Bending Moments and Shears in the Bridge shown below and analyzed previously.
2. Utilize the information from the Grillage Analysis to determine the Distribution Factors for Moment and Shear for exterior and interior Girders for the case of 1-lane and 2-lanes loaded.



Example 6.4 Barker & Puckett

Grillage Analysis

Effective Flange width for interior beams is lesser of:

$$i) \frac{1}{4} \text{ of effective span} = \frac{1}{4} \times 10,668 = 2667 \text{ mm}$$

ii) 12 x Slab depth, plus greater of web thickness
or $\frac{1}{2}$ width of top flange

$$\approx 12 \times 203 = 2436 \text{ mm}$$

$$iii) \text{ c/c spacing of beams} = 2438 \text{ mm}$$

$$\Rightarrow \text{Adopt Effective Flange Width } b_e = \underline{\underline{2438}} \text{ mm}$$

Governs

For Exterior Beam, Effective Flange width may be taken as:

$\frac{1}{2}$ Effective Flange Width of Interior Beam

+ Plus least of:

i) $\frac{1}{8}$ of effective span length

$$\frac{1}{2} \times 2438 + \frac{1}{8} \times 10668 = 2553 \text{ mm}$$

ii) 6 times Avg slab depth

$$\approx \frac{1}{2} \times 2438 + 6 \times 203 = 2437 \text{ mm}$$

iii) Width of the overhang

$$\frac{1}{2} \times 2438 + 997 = \underline{\underline{2216}} \text{ mm}$$

Governs

Example 6.4 Barker & Puckett

Section Properties

Girder Properties

$$E_s = 29000 \text{ ksi} = 200 \times 10^3 \text{ MPa}$$

$$A_g = 31.7 \text{ in}^2 = 20453 \text{ mm}^2$$

$$d = 29.83 \text{ in} = 757.7 \text{ mm}$$

$$e_g = \text{Girder Eccentricity} = \frac{d}{2} + \frac{t_s}{2}$$

$$= \frac{29.83}{2} + \frac{8}{2} = 18.92 \text{ in} = 481 \text{ mm}$$

$$I_g = 4470 \text{ in}^4$$

(Non Composite Girder)

$$= 1860.6 \times 10^6 \text{ mm}^4$$

$$J_g = \text{Torsional Constant}$$

Non Composite Girder

$$= 4.99 \text{ in}^4$$

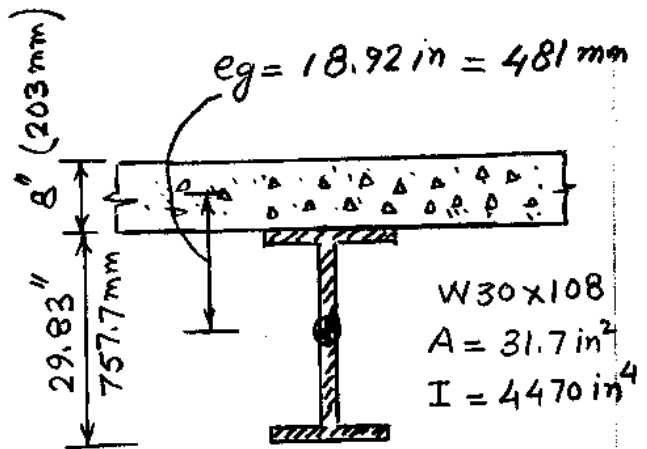
$$= 2.077 \times 10^6 \text{ mm}^4$$

$$I_{gt} \text{ (Composite Girder)} = I_g + A_g e_g^2$$

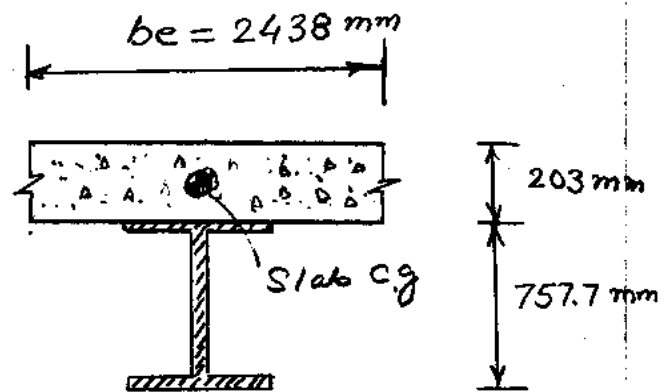
$$= 4470 + 31.7 \times (18.91)^2$$

$$= 15810 \text{ in}^2 \text{ (Steel)}$$

$$= 6580.6 \times 10^6 \text{ mm}^4$$





Example 6.4 Barker & PickettGrillage Element PropertiesLongitudinal ElementInterior Girder

I_c = Moment of Inertia
of Composite Girder

$$= I_s + n I_{gt}$$

I_s = Moment of Inertia of
tributary portion of Slab

$$= b_e \times i^3 = 2438 \times 700,000 = 1706.6 \times 10^6 \text{ mm}^4$$

I_{gt} = Moment of Inertia of
Girder transformed to
Slab Centroid

$$= 6580.6 \times 10^6 \text{ mm}^4$$

n = Modular Ratio

$$= \frac{E_s}{E_c} = \frac{200 \times 10^3}{24830} = 8.05$$

$$\approx \underline{\underline{8.0}} \text{ (say)}$$

$$I_c = 1706.6 \times 10^6 + 8 \times 6580.6 \times 10^6$$

$$= 54351 \times 10^6$$

$$= 54,351 \times 10^6 \text{ mm}^4$$

Example 6.4 Barker & PuckettGrillage Longitudinal Element Props

J_c = Torsional Constant for Composite Girder

$$= J_s + n J_g = b_e j_s + n J_g$$

$$= 2438 \times 1400,000 + 8 \times 2.077 \times 10^6 = 3430 \times 10^6 \text{ mm}^4$$

Exterior Longitudinal Element

$$I_c = b_e \cdot i_s + n I_{gt}$$

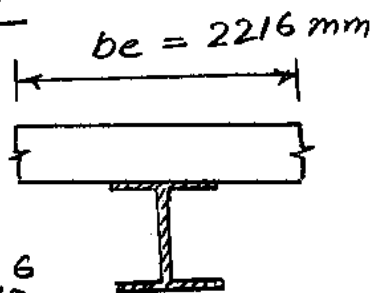
$$= 2216 \times 700,000 + 8 \times 6580.6 \times 10^6$$

$$I_c = 54196 \times 10^6 \text{ mm}^4$$

$$J_c = b_e \cdot j_s + n J_{gt}$$

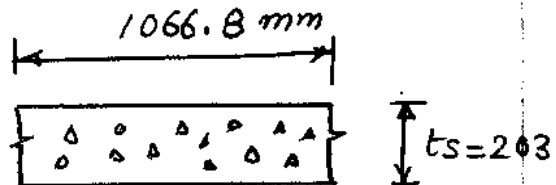
$$= 2216 \times 1400,000 + 8 \times 2.077 \times 10^6$$

$$= 3119 \times 10^6 \text{ mm}^4$$



Example 6.4 Barker & Pockett

Grillage Transverse Element Properties



$$I_c = b_e \cdot i_s$$

$$= 1066.8 \times 700,000$$

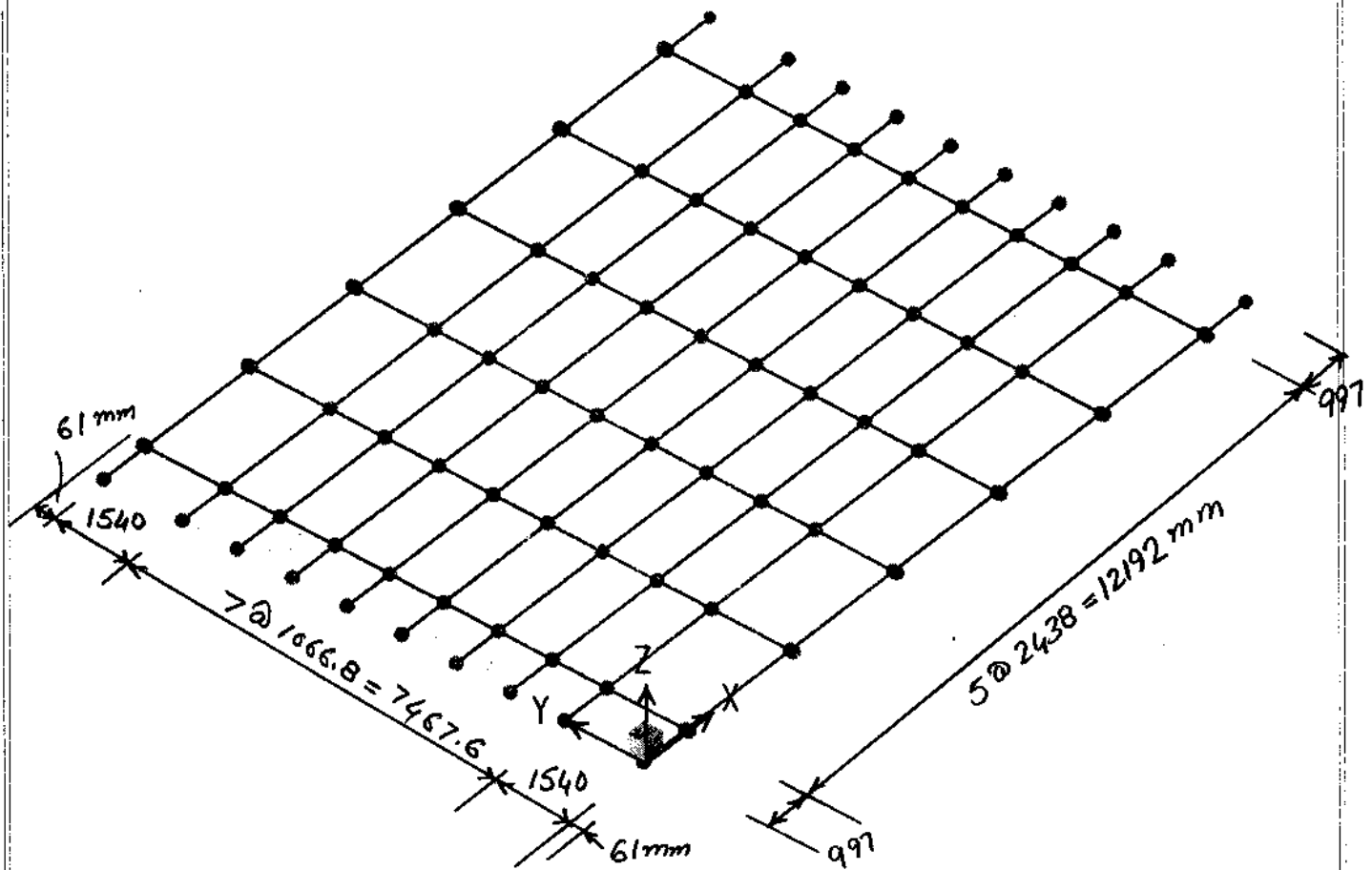
$$= 746.8 \times 10^6 \text{ mm}^4$$

$$J_c = b_e \cdot j_s$$

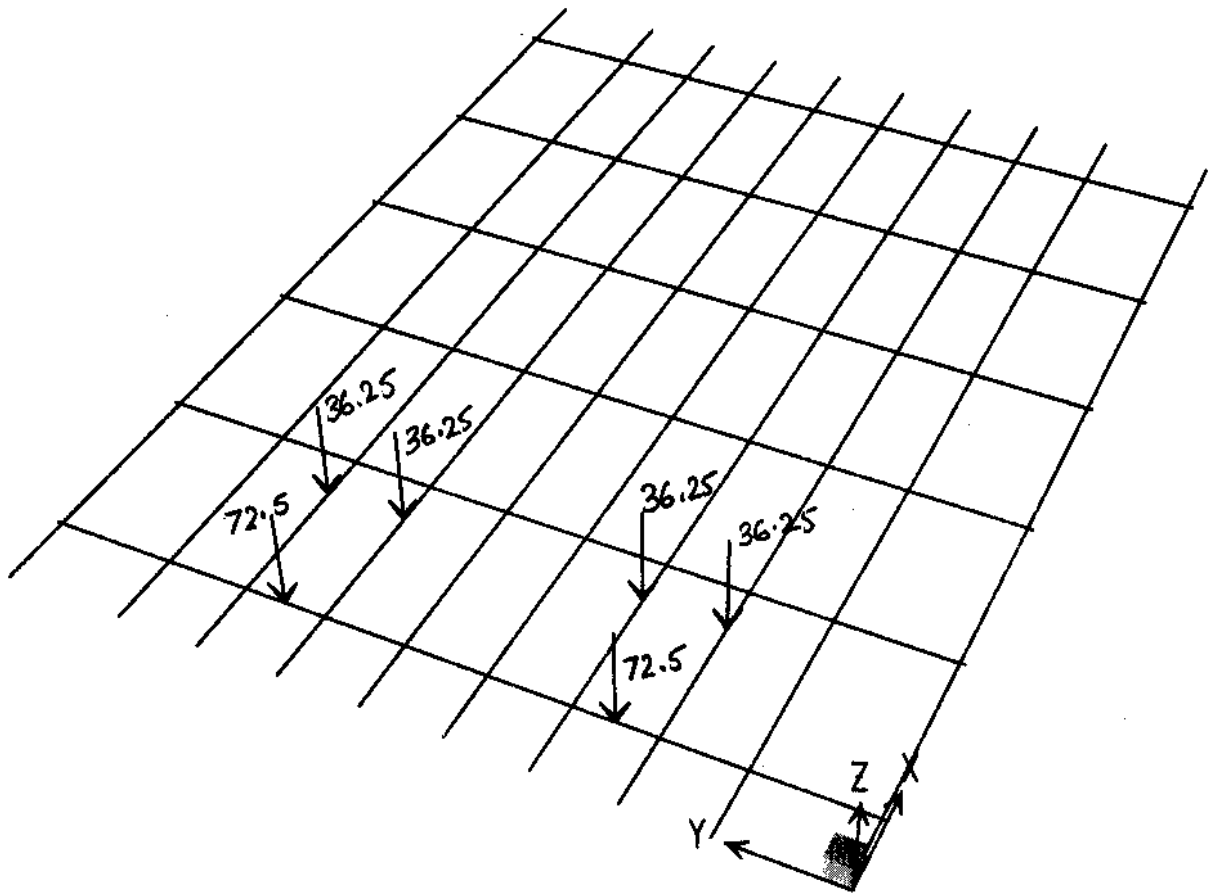
$$= 1066.8 \times 1400,000$$

$$= 1493.5 \times 10^6 \text{ mm}^4$$

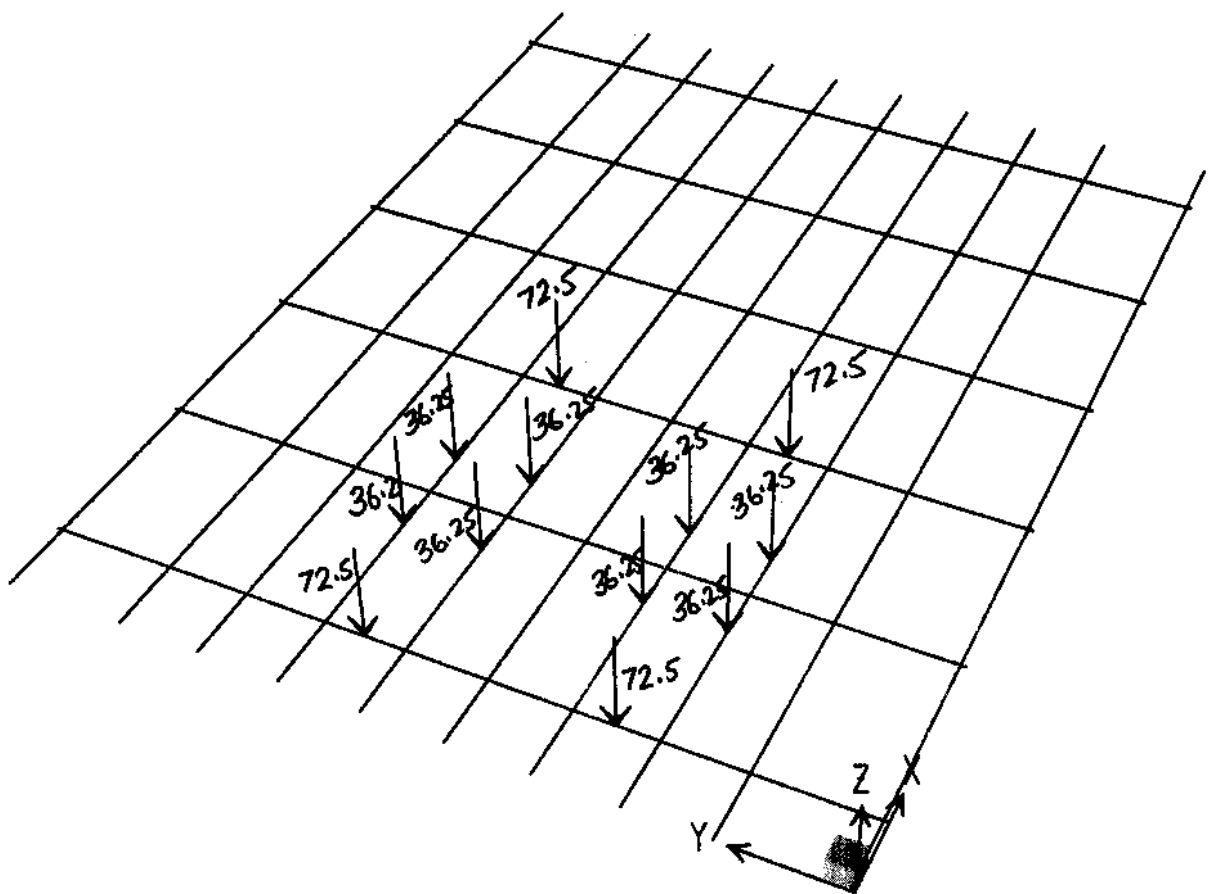
21



FINITE ELEMENT GRILLAGE MESH

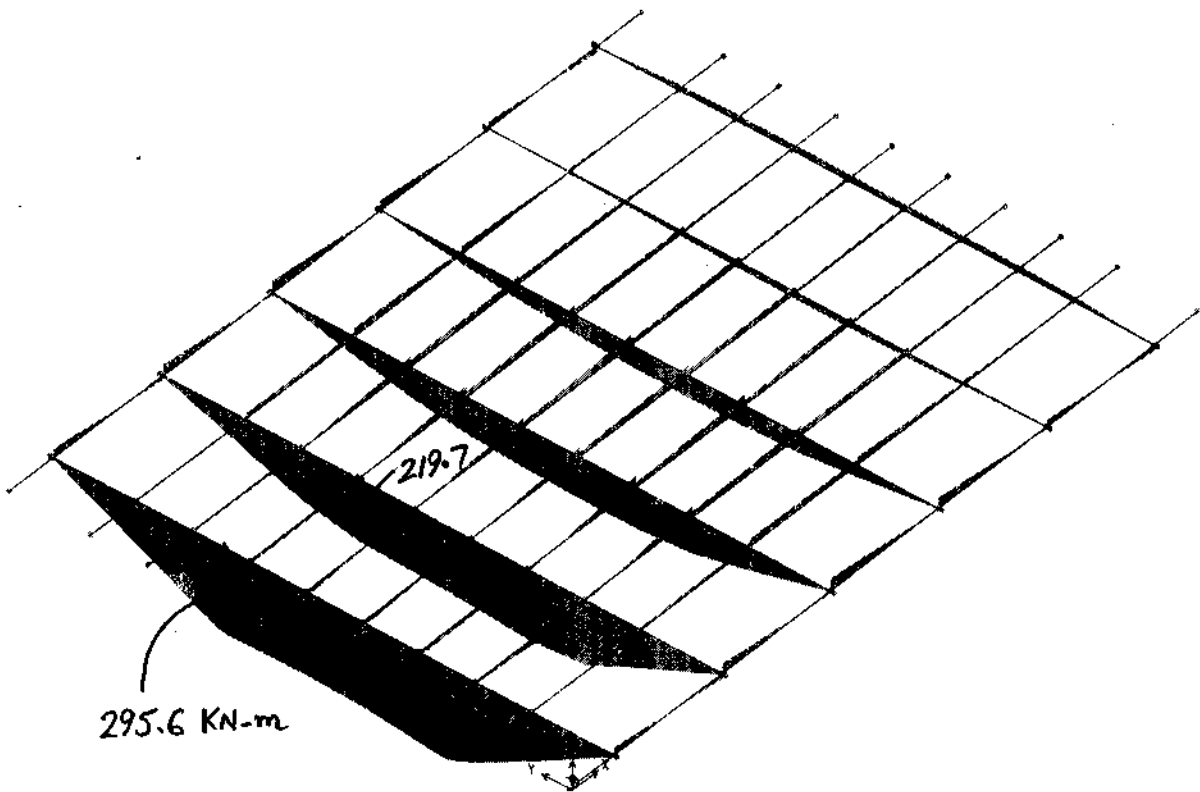


LOAD POSITIONS & VALUES
FOR SINGLE LANE LOADING.



LOAD POSITIONS & VALUES
FOR 2-LANES LOADED

BENDING MOMENTS / DISTRIBUTION FACTORS
SINGLE LANE LOADING.



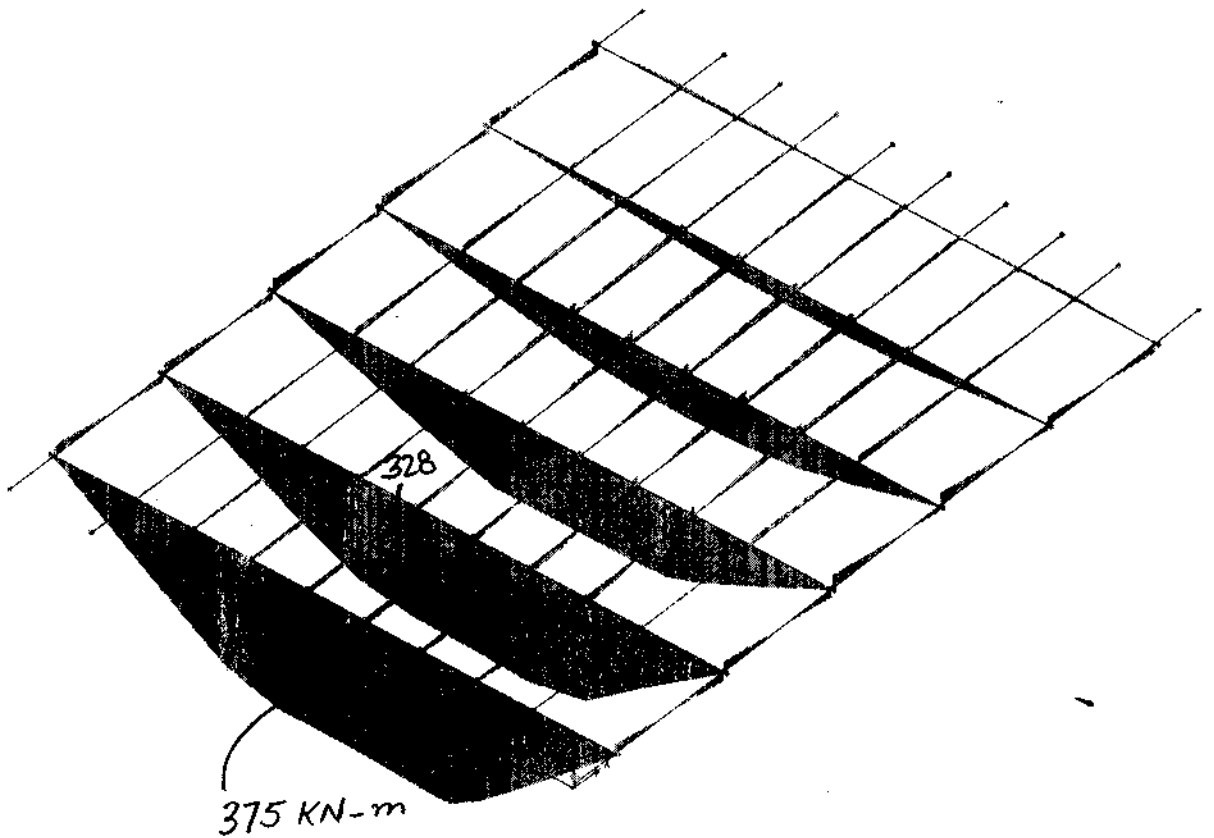
$$\text{BEAM MOMENT} = 498.7 \text{ KN-m}$$

$$m_{\text{moment}}^{\text{SE}} = \frac{295.6}{498.7} = 0.59$$

$$m_{\text{moment}}^{\text{SI}} = \frac{219.7}{498.7} = 0.44$$

25

BENDING MOMENTS / DISTRIBUTION FACTORS
 2-LANES LOADED.



$$\text{BEAM MOMENT} = 498.7 \text{ KN-m}$$

$$m_{\text{moment}}^{mE} = \frac{375}{498.7} = 0.75$$

$$m_{\text{moment}}^{mI} = \frac{328}{498.7} = 0.66$$