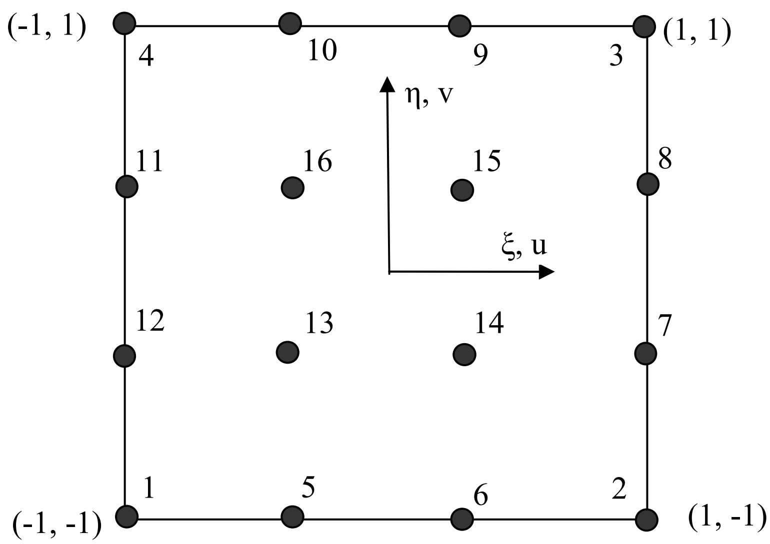
CE-5155
Finite Element Analysis of Structural Systems
Final Exam, Fall 2009

Take Home Exam.
Due Date: 10 March 2010
Maximum Marks: 100

Question. No. 1. (15 Marks)

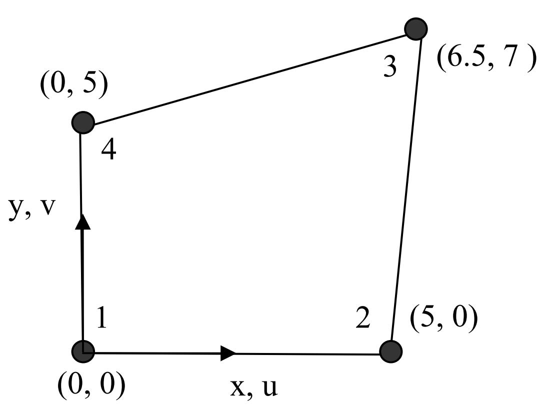


a) Determine the shape function for Node 4 in Natural Coordinates: $N_4(\xi, \eta)$ (5 marks)

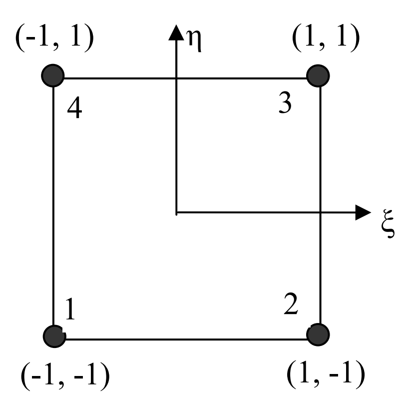
b) Plot the value of u displacement along edge 4-10-9-3 for a set of prescribed nodal displacements: $u_4 = 1.0$, $v_4 = 2.0$ and all other nodes $u = v = 0.0$ (5 marks)

c) Plot ε_x along edge 4-10-9-3 for the prescribed displacements in part b. (5 marks)

Question. No. 2. (25 Marks)



Actual Element



Parent Element

A 4-noded plane stress element is shown above. The nodal coordinates are given in parenthesis.

Evaluate the following at $(\xi, \eta) = (0, 0)$:

a) (x, y) (5 marks)

b) $[\,J\,]_{2\times 2}$, the Jacobian Matrix of the Transformation that establishes the relationship of the following form between the natural coordinates and the actual coordinates: (5 marks)

$$\begin{Bmatrix} \dfrac{\partial}{\partial \xi} \\ \dfrac{\partial}{\partial \eta} \end{Bmatrix} = [J] \begin{Bmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \end{Bmatrix}$$

c) $[\,J\,]^{-1}_{2x2}$, The Inverse of Jacobian Matrix (5 marks)

d) Det [J] (5 marks)

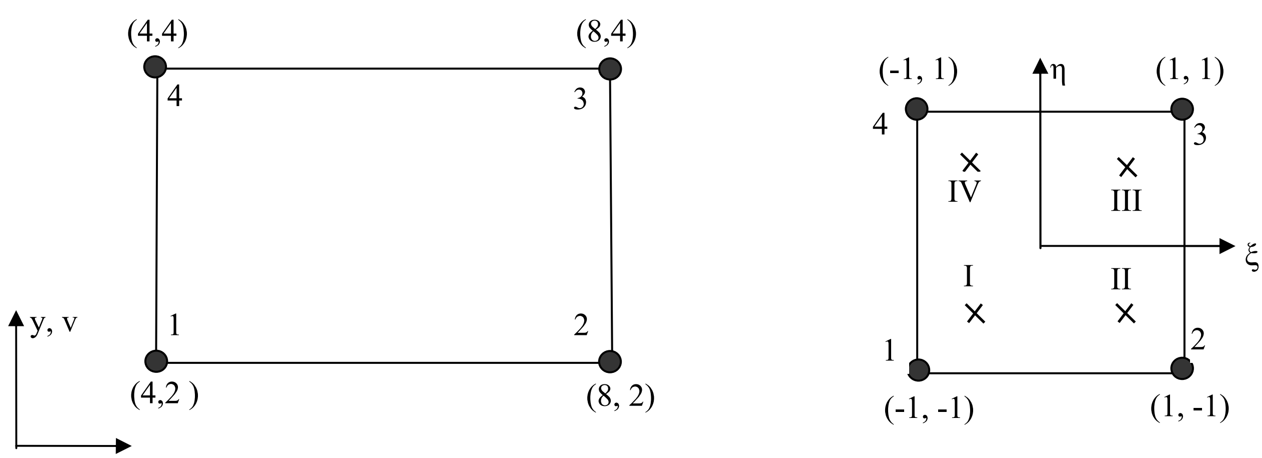
e) [B] , The Strain-Displacement Matrix (5 marks)

Hint: Start with relations:

$$x = \sum_{i}^{n} N_i(\xi,\eta)\hat{X}_i$$

$$y = \sum_{i}^{n} N_i(\xi,\eta)\hat{Y}_i \text{ , where } \hat{X}_i \text{ , } \hat{Y}_i \text{ are nodal coordinates}$$

Question. No. 3. (25 Marks)



Actual Element

Parent Element

Integrate the function f (x, y) = x + y^2 + xy + 1/x over the PSR element shown above. Use 2x2 Gauss quadrature to perform the integration. Follow the steps outlined below:

a) Establish a relation between the actual coordinates x, y and the natural coordinates ξ, η. (5 marks)

b) Transform the function f (x,y) into function f(ξ, η) in natural coordinates.(5 marks)

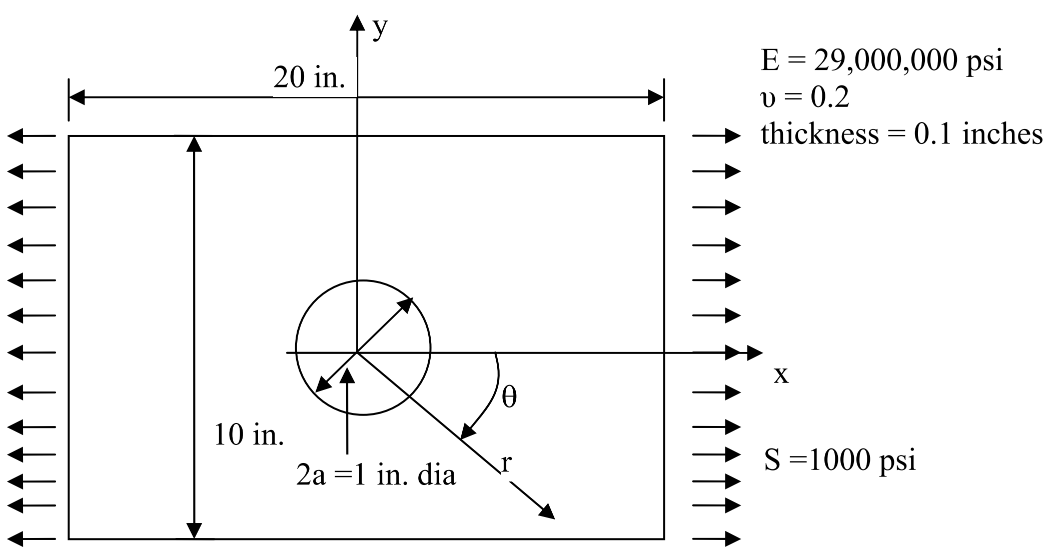
c) Find the determinant of Jacobian Matrix Transformation between the parent element and the actual element such that **dx dy = Det | J | dξ dη.** (5 marks)

d) Evaluate the Integral the following integral using a 2x2 Gauss rule: (5 marks)

$$I = \int_{y=2}^{4} \int_{x=4}^{8} f(x, y)\, dx\, dy$$

e) Is the evaluated integral exact? Briefly explain (5 marks)

Question. No. 4. (35 Marks)



A rectangular plate shown above has a hole in its center with pressure applied in the longitudinal direction. The dimensions of the plate and hole and the pressure intensity is shown in the figure.

a) Make a suitable mesh using SAP2000 to analyze the problem and to capture the stress gradients. Utilize symmetry of the problem to make the Finite Model. You should have appropriate mesh refinement where needed only as points will be awarded for optimum mesh configuration. Indicate the pressures applied and boundary conditions used in modeling. You can use the 2-Dimensional Asolid or Shell/Membrane Element to analyze the problem.

(10 marks)

b) Refine the mesh so that numerical results are within 5% of the exact solution. (5 marks)

For Part (b) and (c) make use of the exact theory of elasticity solution to the problem that is given below for reference:

$$\sigma_r = \frac{S}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{S}{2}\left(1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right)\cos 2\theta$$

$$\sigma_\theta = \frac{S}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{S}{2}\left(1 + \frac{3a^4}{r^4}\right)\cos 2\theta$$

$$\tau_{r\theta} = -\frac{S}{2}\left(1 - \frac{3a^4}{r^4} + \frac{2a^2}{r^2}\right)\sin 2\theta$$

c) The Plot the numerically obtained Tangential Stress (σ_θ) variation on y-y axis and compare to the exact solution.

(10 marks)

d) Plot the Radial Stress (σ_r)variation along x-x axis and compare to the exact solution

(10 marks)