CE-5155
Finite Element Analysis of Structural Systems
Midterm Exam, Fall 2009

Take Home Exam
Due Date: December 26, 2009 (No Exceptions)
Maximum Marks 150

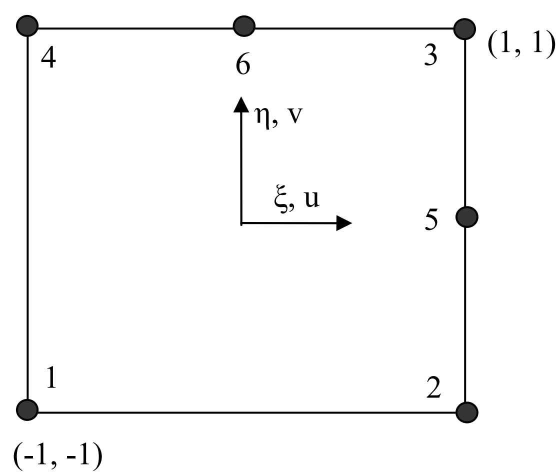
Note: You are expected to attempt the exam independently. No joint effort is allowed. In case of sharing of answers or joint effort, no credit will be given.

Question. No. 1. Answer the following True/ False Questions (2 pts each = 40 marks)

_______________	a.	The principle of stationary potential energy is used to enforce equilibrium and displacement boundary conditions in structural mechanics problems
_______________	b.	The completeness requirement for conforming finite element specifies that an element must include all possible constant strain modes, but need not model linear strain variations.
_______________	c.	The governing equation of equilibrium is satisfied exactly in finite element stiffness equations, but displacement compatibility is only approximated.
_______________	d.	The natural boundary conditions on stress are approximated in Galerkin Method.
_______________	e.	For plane stress displacement finite elements the shape functions must sum to unity (1.0) at all the nodes as well as everywhere in the element.
_______________	f.	The deflections obtained by using a conforming and a complete finite element will always be more than the exact solution.
_______________	g.	The principle of minimum potential energy and the principle of virtual work are essentially one and the same thing.
_______________	h.	If a supercomputer is used to generate highly refined meshes an exact solution for all structural mechanics problems can be obtained.
_______________	i.	Body forces are forces that are distributed over a surface area of a body.
_______________	j.	In a finite element solution the stresses can be discontinuous across element boundaries.

________________	k.	Conforming finite elements at least must be able to represent both constant and linear distributions of strain.
________________	l.	In conforming finite elements the stresses are compatible at element boundaries.
________________	m.	A finite element formulation based on minimization of potential energy and another formulation based on the principal of virtual work may yield slightly different results.
________________	n.	Numerical integration of polynomials by Newton Cotes Formulas or Gauss Integration Formulas yield fairly accurate results but are never exact.
________________	o.	The essential boundary conditions don't have to be exactly satisfied if a Weighted Residual Formulation for finite element solution is being used.
________________	p.	In isoparametric elements, the element we have a choice to formulate element stiffness matrices either by exact integration formulas or by numerical integration.
________________	q.	Use of Hermetian Shape Functions in beam elements ensures continuity of displacements and shear forces across element boundaries.
________________	r.	An axisymmetric problem can be solved by using a plane strain element.
________________	s.	Finite element solutions based on Galerkin method and on Minimization of Potential Energy would always yield similar but different results.
________________	t.	In a finite element, the sum of shape functions is equal to 1.0 at the nodes only.

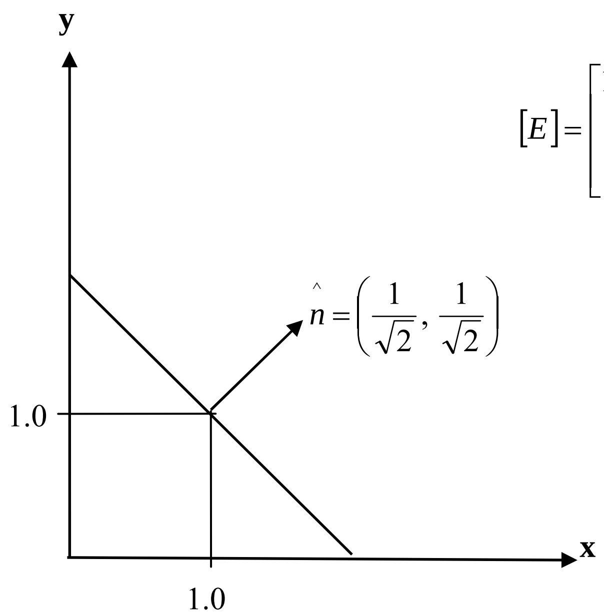
Question. No. 2. (20 Marks)



Develop the shape functions for the above shown element

Hint: Start from shape functions for a 4-noded Q4 element

Question 3. (20 Marks)



$$[E] = \begin{bmatrix} 1000 & 300 & 0 \\ 300 & 1000 & 0 \\ 0 & 0 & 100 \end{bmatrix} ksi$$

The strain field in a triangular region bounded by x = 0, y = 0, x + y = 2.0 is assumed to be of the form:

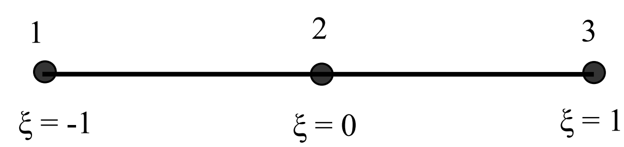
$$\varepsilon_{xx} = a_1 x + a_2 y$$
$$\varepsilon_{yy} = b_1 x + b_2 y$$
$$\gamma_{xy} = c_1 x + c_2 y$$

With body forces Fx = d = constant and Fy = 0.0

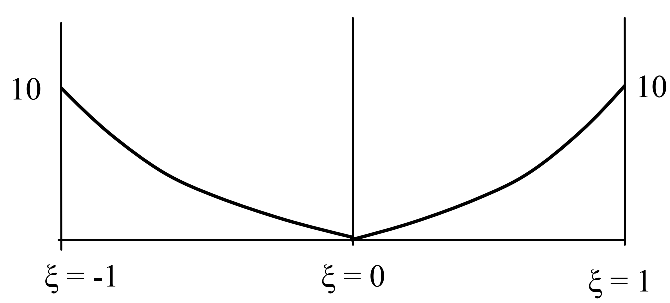
a) Use the stress-strain matrix to compute σ_{xx}, σ_{yy}, τ_{xy} and the stress component in the normal direction σ_{nn}, all at location (x, y) = (1.0, 1.0).

b) Write an equation in a1, a2, b1, b2, c1, c2, and d to guarantee equilibrium in the x direction.

Question. No. 4. (20 Marks)



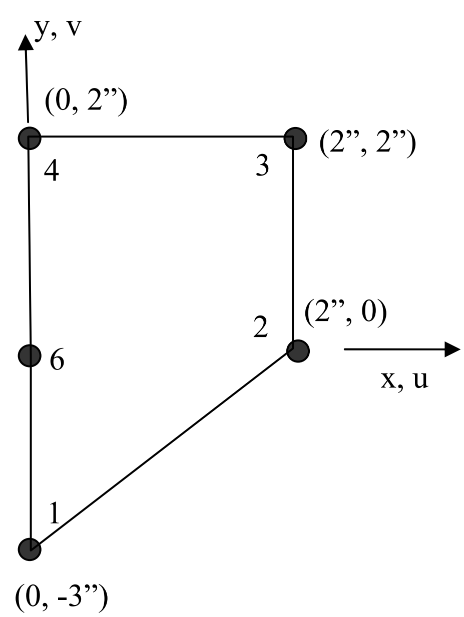
3-noded Rod Element

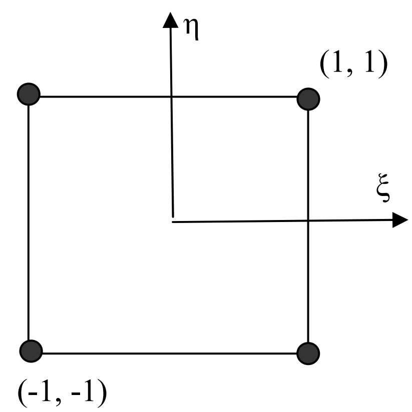


Variation of Axial force $10\,\xi^2$ along the length

The 3-noded bar element shown above is acted upon by an axial distributed force along its length in the X-direction given by 10 ξ^2. Find the work-equivalent nodal force for node 2 using the shape functions for a 3-noded bar element. **Use of 3 point Gauss Numerical Quadrature for your computations is compulsory.**

Question. No. 4. (50 Marks)





A 4-noded isoparametric plane stress element is shown above. The thickness of the element is 2/3 inches. The element nodal displacement vector is (u_1, v_1, ….. u_4, v_4)and the vector of strain components is (ε_{xx}, ε_{yy}, γ_{xy})

a) Compute the following quantities at the location (ξ, η) = (0, 0) (40 Marks)

i. $[J]_{2x2}$, where [J] is a 2x2 Jacobian Matrix

ii. Det [J], i.e. determinant of the Jacobian Matrix

iii. $[J]^{-1}$, inverse of Jacobian Matrix

iv. $[B]_{3x8}$, 3x8 Strain-Displacement Matrix

b) Use a 1 x 1 Gauss Integration Formula to calculate the approximate volume of the element. (10 Marks)