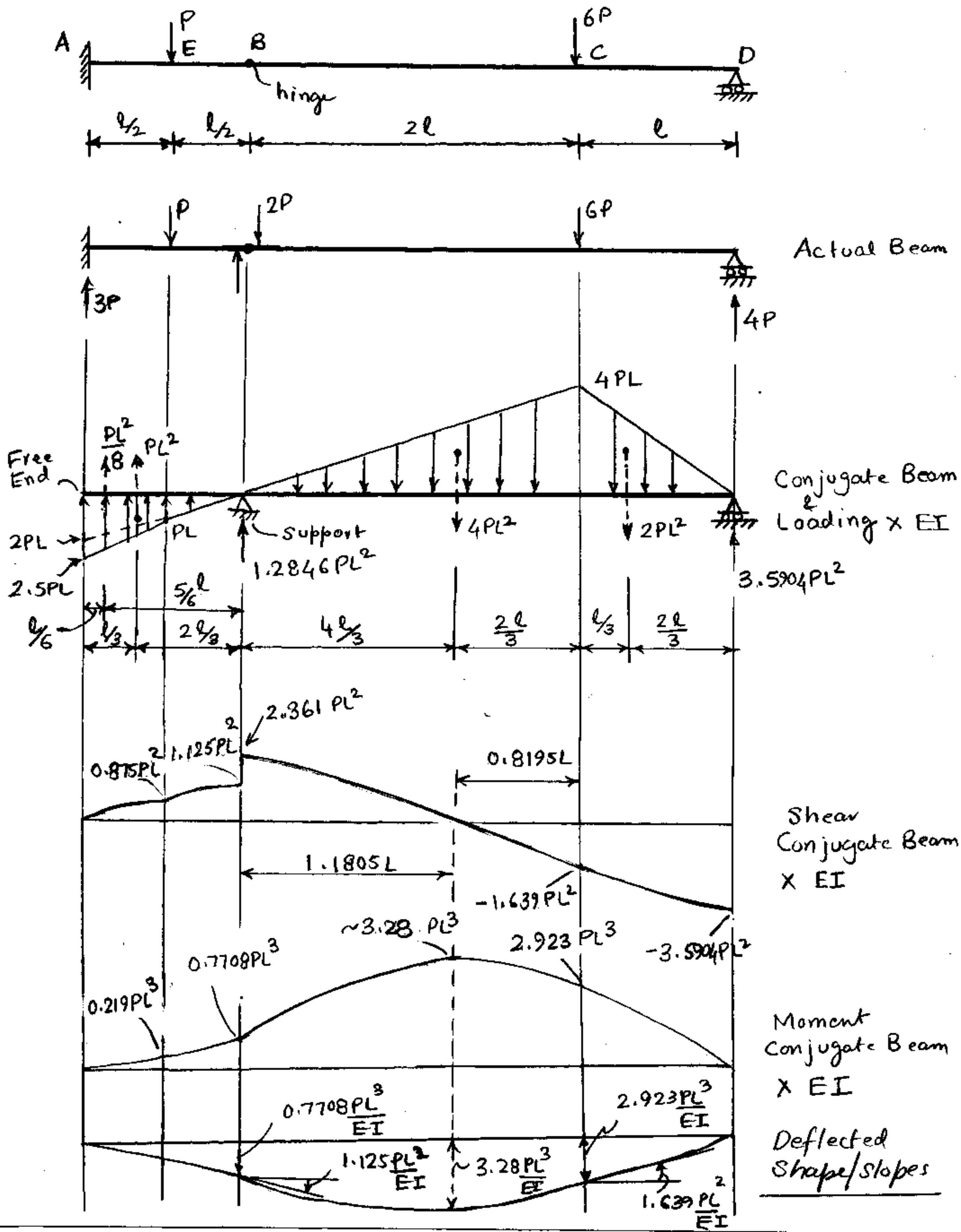
Solution Assignment #1

Q.1 Find the slopes and deflections of the beam shown below at critical locations using conjugate beam method. Draw a neat sketch of the deformed shape showing displacements and slopes.



Ans.

Reaction @ D

Taking moment @ Pt B

$$R_D \times 3\ell = 6P \times 2\ell$$

$$R_D = \frac{12P}{3} = 4P \uparrow$$

$$\Rightarrow R_A = 7P - R_D = 7P - 4P = 3P \uparrow$$

$$\Rightarrow R_B = 6P - R_D = 6P - 4P = 2P \uparrow$$

$$M_A = 2P \times \ell + P \times \ell/2$$

$$M_A = 2PL + \frac{PL}{2} = \frac{5}{2}PL$$

Areas of Loads on Conjugate Beam:

$$\text{Area } CD = 4PL \times \frac{\ell}{2} = 2PL^2$$

$$\text{Area } BC = 4PL \times \frac{2\ell}{2} = 4PL^2$$

$$\text{Area } AB = 2P \times \frac{\ell}{2} = PL^2$$

$$\text{Area } AE = 0.5PL \times \frac{\ell}{2} \times \frac{1}{2} = \frac{PL^2}{8}$$

Conjugate Beam Reactions

Taking moments @ D

$$R_B \times 3\ell + \frac{PL^2}{8} \times \left(\frac{5}{6}\ell + 3\ell\right) + PL^2 \times \left(\frac{2\ell}{3} + 3\ell\right)$$

$$- 4PL^2 \times \left(\frac{2\ell}{3} + \ell\right) - 2PL^2 \times \frac{2\ell}{3} = 0$$

$$R_B \times 3\ell + 0.4792\ PL^3 + 3.6667\ PL^3 - 6.6667PL^3 - 1.333PL^3 = 0$$

$$R_B \times 3\ell = 3.8538PL^3$$

$$\Rightarrow R_B = \frac{3.8538 \times PL^3}{3\ell} = 1.2846PL^2 \uparrow$$

Ans. 1

$$R_D = 4PL^2 + 2PL^2 - \frac{PL^2}{8} - PL^2 - 1.2846PL^2$$

$$R_D = 3.5904\,PL^2 \uparrow$$

Shear @ Left of B = Slope left of B

$$\frac{PL^2}{8} + PL^2 = 1.125\,PL^2$$

Divide By EI

Shear @ Left of B = Slope Left of B $= 1.125\frac{PL^2}{EI}$ clockwise

Shear @ Right of B = Slope Right of B

$$= 1.125PL^2 + 1.2846PL^2 = 2.4096PL^2$$

$\Rightarrow$ Slope Right of B = $= 2.4096\frac{PL^2}{EI}$ Clockwise

Shear @ Pt C

Shear @ Pt C $= 2.4096PL^2 - 4PL^2 = -1.5904PL^2$

$= -1.5904\frac{PL^2}{EI}$ anticlockwis

Shear @ Pt D

$= -1.5904PL^2 - 2PL^2 = -3.5904PL^2 = R_D$ (Check) ✓

$$\text{Moment @ } B = \frac{PL^2}{8} \times \frac{5}{6} l + PL^2 \times \frac{2l}{3} = 0.7708\, PL^3$$

$$= 0.7708 \frac{PL^3}{EI}$$

$$\text{Moment @ } D = \frac{PL^2}{8} \times \left(\frac{5}{6} l + 3l\right) + PL^2 \times \left(\frac{2l}{3} + 3l\right)$$

$$+ 1.2846\, PL^2 (3l) - 4PL^2\left(\frac{2}{3} l + l\right) - 2PL^2\left(\frac{2l}{3}\right)$$

$$= 0.479\, PL^3 + 3.6667\, PL^3 + 3.8538\, PL^3$$

$$- 6.6667\, PL^3 - 1.333\, PL^3 = 0 \quad \text{(zero)}$$

Check ✓

$$\text{Moment @ } C = 3.5904\, PL^2 \times l - 2\, PL^2 \times \frac{l}{3} = 2.9237\, PL^3$$

From other end

$$\text{Moment @}C = \frac{PL^2}{8}\left(\frac{5}{6} l + 2l\right) + PL^2\left(\frac{2l}{3} + 2l\right)$$

$$+ 1.2846\, PL^2 (2l) - 4PL^2\left(\frac{2l}{3}\right)$$

$$= (0.35416 + 2.6667 + 2.5692 - 2.6667)\, PL^3 = 2.923\, PL^3$$

(Check) ✓

$$\text{Moment @ } C = \text{Deflection @ } C = 2.923 \frac{PL^3}{EI} \downarrow$$

Moment @ $\frac{2}{3} l$ left of C by summation of shear Diagram

From Right support

$$= \frac{(3.5904 + 1.639)}{2} PL^3 + \frac{1.639 \times 0.8195}{2} PL^3$$

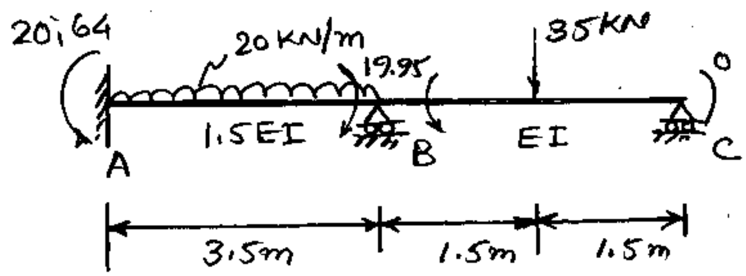
$$= 3.28\, PL^3 \qquad = 3.28\, PL^3$$

$$= 3.28 \frac{PL^3}{EI}$$

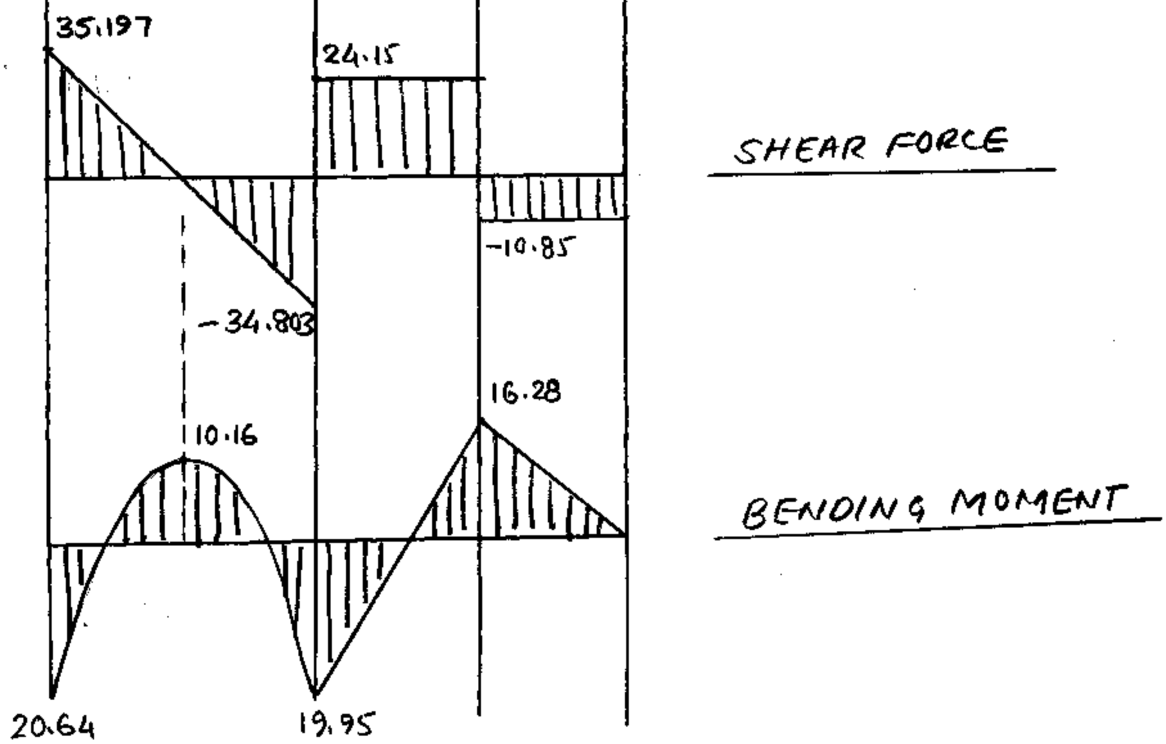
Answer

Assignment # 1

Q No. 2 Draw the Bending Moment and shear force diagram of the Beam shown below using slope-deflection method.

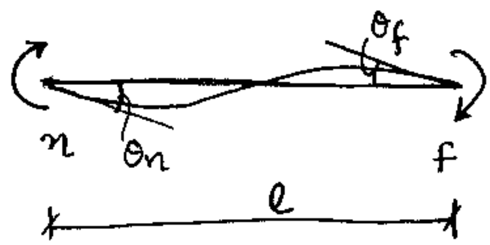


R Load	+35	+35	+17.5	+17.5
R Moment	+0.197	-0.197	+6.65	-6.65
Total	+35.197	+34.803	+24.15	+10.85



Assignment # 1

Q No. 2



$$M_{nf} = 2EK_{nf}\,(2\theta_n + \theta_f) + FEM_{nf}$$

$$FEM_{AB} = -\frac{wl^2}{12} = -\frac{20 \times (3.5)^2}{12} = -20.41 \text{ KN-m}$$

$$FEM_{BA} = +\frac{wl^2}{12} = \quad = 20.41 \text{ KN-m}$$

$$FEM_{BC} = -\frac{Pab^2}{l^2} = \frac{-35 \times 1.5 \times (1.5)^2}{(3)^2} = -13.12 \text{ KN-m}$$

$$FEM_{CB} = \quad = +13.12 \text{ KN-m.}$$

$$K_{AB} = \frac{1.5\,I}{3.5} = 0.4286\,I = 1.286\,K$$

$$K_{BC} = \frac{I}{3} = 0.3333\,I = K$$

<u>Slope-Deflection Equations</u>

$$M_{AB} = 2E\,(1.286K)\,(2\theta_A + \theta_B) - 20.41$$
$$M_{BA} = 2E\,(1.286K)\,(2\theta_B + \theta_A) + 20.41$$
$$M_{BC} = 2EK\,(2\theta_B + \theta_C) + 13.12$$
$$M_{CB} = 2EK\,(2\theta_C + \theta_B) + 13.12$$

<u>Boundary Conditions</u>

$\theta_A = 0$,

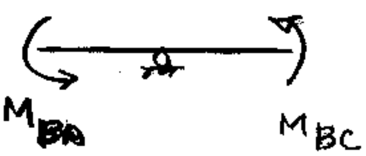
$M_{CB} = 0$

Assignment # 1

Q. No. 2 Substituting BCs in the Slope - Deflection Eqns we have

$$\left.\begin{aligned} M_{AB} &= 2.572\,EK\,\theta_B - 20.41 \\ M_{BA} &= 5.144\,EK\,\theta_B + 20.41 \\ M_{BC} &= 4EK\,\theta_B + 2EK\,\theta_C - 13.12 \\ M_{CB} &= 2EK\,\theta_B + 4EK\theta_C + 13.12 \end{aligned}\right\} \text{— (1)}$$

Equilibrium @ Joint B & BC @ End C



$$\left.\begin{aligned} M_{BA} + M_{BC} &= 0 \\ M_{CB} &= 0 \end{aligned}\right\} \text{— (2)}$$

Substituting Eqns (1) in Eqn (2) we have

$$M_{BA}+M_{BC} = 9.144\,EK\,\theta_B + 2EK\theta_C + 7.29 = 0$$

$$M_{CB} = 2EK\,\theta_B + 4EK\theta_C + 13.12 = 0$$

$$\begin{bmatrix} 9.144 & 2 \\ 2 & 4 \end{bmatrix} \begin{Bmatrix} EK\theta_B \\ EK\theta_C \end{Bmatrix} = \begin{Bmatrix} -7.29 \\ -13.12 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} EK\theta_B \\ EK\theta_C \end{Bmatrix} = \begin{matrix} -0.08964 \\ -3.23518 \end{matrix}$$

$$M_{AB} = 2.572(-0.08964) - 20.41 = -20.64 \text{ kN-m}$$

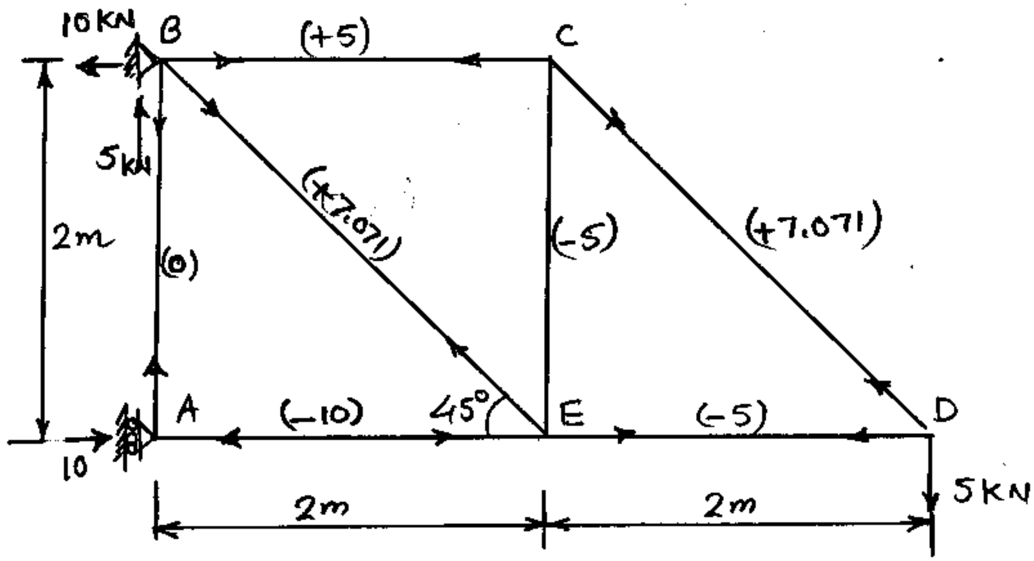
$$M_{BA} = 5.144(-0.08964) + 20.41 = +19.95 \text{ kN-m.}$$

Solution Assignment # 2

Q. No. 1 Find Vertical and horizontal displacements of Pt D in the truss shown below.

$A = 1000\ mm^2$, $E = 200\ GPa = 200 \times 10^3\ N/mm^2 = 200\ KN/mm^2$

Use method of Virtual Work.



Take moment at B

$R_{BH} \times 2 = 5 \times 4 \Rightarrow R_{BH} = \frac{20}{2} = 10\ KN$

$F_{CD} \sin 45 = 5\ KN \Rightarrow F_{CD} = \frac{5}{\sin 45} = +7.071\ KN$

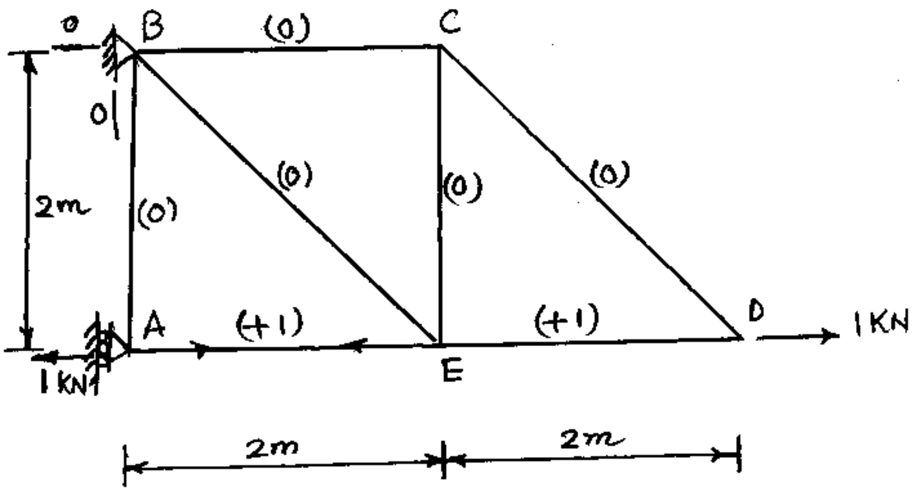
$F_{ED} + F_{CD} \cos 45° \Rightarrow F_{ED} = -F_{CD} \cos 45° = -5.0\ KN$

$F_{BC} = F_{CD} \cos 45 \Rightarrow F_{BC} = F_{CD} \cos 45 = +5.0\ KN$

* The Forces for a unit Vertical load @ pt D would be equal to 1/5 times the above listed forces.

Solution Assignment # 2

For horizontal Displacement @ pt D we apply a horizontal unit load @ pt D



By inspection $R_{AH} = 1 KN \leftarrow$

$\Rightarrow$ $R_{BH} = 0$

Virtual Work Principal

$$1 \cdot \Delta_{DV} = \sum_{j=1}^{m} \frac{F \cdot f \cdot L}{AE}$$

F = Primary Structure Force

f = Force due to Unit Load.

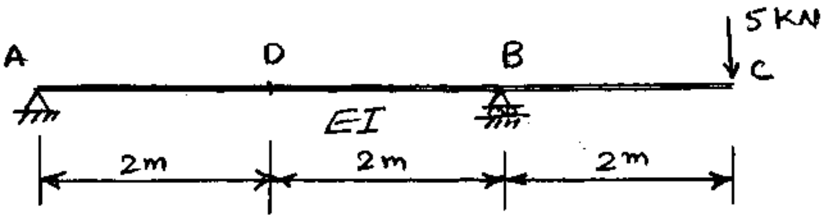
Member	F (KN)	L (m)	f Δv (KN)	f Δh (KN)	F.fΔv.L KN.m	F.fΔh.L (KN-m)
AB	0	2	0	0	—	—
BC	+5	2	1	0	10	—
AE	−10	2	−2	+1	40	−20
BE	+7.071	2.828	+1.4142	0	28.279	—
CE	−5	2.	−1	0	10	—
ED	−5	2	−1	+1	10	−10
CD	+7.071	2.828	+1.4142	0	28.279	—
				Σ	126.56	−30

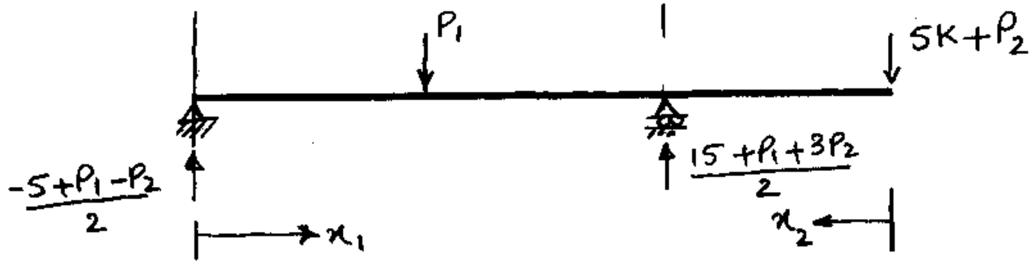
$$1 \cdot \Delta_{DV} = \sum \frac{F \cdot f \cdot L}{AE} = \frac{126.56\ (KN\text{-}m) \times 1000}{1000 \times 200} = 0.6328 mm \downarrow$$

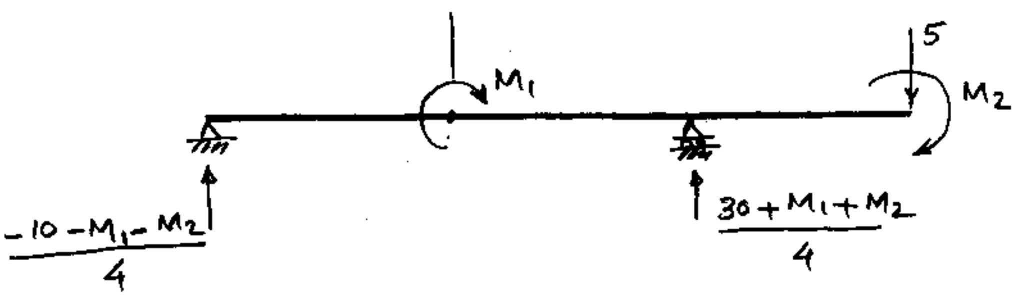
$$1 \cdot \Delta_{DH} = \sum \frac{F f l}{AE} = \frac{-80 \times 1000}{1000 \times 200} = -0.15 mm \leftarrow$$

Solution Assignment # 2

Q No. 2 For the beam shown below find the deflections and slopes at pts C & D. Use Castigliano's Theorum for solution. Take EI = constant for the beam







Solution Assignment #2

Q. No. 2

Castigliano's Second Theorum states that

$$\frac{\partial U}{\partial P_i} = \Delta_i$$

$$U = \text{Strain Energy} = \begin{array}{c}\text{Strain Energy}\\ \text{of Bending}\end{array} = \int \frac{M^2}{2EI}\,dx$$

$$\frac{\partial U}{\partial P_i} = \int \frac{1}{EI} M \cdot \frac{\partial M}{\partial P_i}\,dx$$

To Find Deflections @ C & D we impose loads P_1 & P_2 imaginary loads @ C & D respectively. Then after applying Castigliano's Second theorem we can substitute P_1 & $P_2 = 0$ to get deflections corresponding to 5K load.

Taking moment @ A

$$2P_1 - 4R_B + (5+P_2)\times 6 = 0$$

$$2P_1 - 4R_B + 30 + 6P_2 = 0 \implies R_B = \frac{30 + 2P_1 + 6P_2}{4}$$

$$R_B = \frac{15 + P_1 + 3P_2}{2} \uparrow$$

$$R_A = 5 + P_1 + P_2 - \frac{15 + P_1 + 3P_2}{2}$$

$$= \frac{10 + 2P_1 + 2P_2 - 15 - P_1 - 3P_2}{2}$$

$$= \frac{-5 + P_1 - P_2}{2} \implies R_A = \frac{-5 + P_1 - P_2}{2}$$

Range $2 \geq x \geq 0$

$$M = \frac{-5 + P_1 - P_2}{2} \cdot x \quad , \quad \frac{\partial M}{\partial P_1} = \frac{x}{2}$$

$$\frac{\partial M}{\partial P_2} = -\frac{x}{2}$$

$$M\frac{\partial M}{\partial P_1} = \frac{-5 + P_1 - P_2}{4} \cdot x^2$$

$$M\frac{\partial M}{\partial P_2} = -\frac{5 + P_1 + P_2}{4}\, x^2$$

Solution Assignment #2

Q No. 2

Range $2 \leqslant x \leqslant 4$

$$M = \frac{5 + P_1 - P_2}{2} \cdot x - P_1 (x-2)$$

$$\frac{\partial M}{\partial P_1} = \frac{x}{2} - (x-2) = -\frac{x}{2} + 2$$

$$\frac{\partial M}{\partial P_2} = -\frac{x}{2}$$

Range $4 \leqslant x \leqslant 6$, $2 \geqslant x_2 \geqslant 0$

$$M = -(5 + P_2)(x_2)$$

$$\frac{\partial M}{\partial P_1} = 0$$

$$\frac{\partial M}{\partial P_2} = -x_2$$

$$M \frac{\partial M}{\partial P_1} = \frac{5 - P_1 + P_2}{2} \left(\frac{x^2}{2} - 2x\right) + \frac{P_1}{2} (x-2)\left(-\frac{x}{2} + 2\right)$$

$$M \frac{\partial M}{\partial P_2} = \frac{5 - P_1 + P_2}{4} x^2 + \frac{P_1}{2} (x^2 - 2x)$$

$$M \frac{\partial M}{\partial P_1} = 0$$

$$M \frac{\partial M}{\partial P_2} = (5 + P_2)\, x_2^2$$

Apply Castigliano's 2nd Theorum

$$\frac{\partial U}{\partial P_1} = \sum \int \frac{1}{EI} M \cdot \frac{\partial M}{\partial P_1} \cdot dx = D_1$$

* We Now substitute $P_1 = P_2 = 0$ as these forces actually do not exist

$$EI \frac{\partial U}{\partial P_1} = \int_0^2 -\frac{5}{4} x^2 + \int_2^4 \frac{5}{2} \left(\frac{x^2}{2} - 2x\right)$$

$$= -\frac{5}{4} \left|\frac{x^3}{3}\right|_0^2 + \frac{5}{2} \left|\frac{x^3}{6} - x^2\right|_2^4$$

$$= -\frac{5}{4}\left(\frac{8}{3}\right) + \frac{5}{2}\left[\frac{4^3}{6} - 4^2 - \frac{2^3}{6} + 2^2\right]$$

$$= -\frac{40}{12} + \frac{5}{2}\left[\frac{64}{6} - 16 - \frac{8}{6} + 4\right]$$

$$= -\frac{40}{12} + \frac{5}{2}\left[\frac{32}{3} - 16 - \frac{4}{3} + 4\right]$$

$$= -\frac{40}{12} + \frac{5}{2}\left[\frac{32 - 48 - 4 + 12}{3}\right]$$

Solution - Assignment # 2

QNo. 2

$$EI\frac{\partial U}{\partial P_1} = \frac{-40}{12} + \frac{5}{2}\left(\frac{-8}{3}\right) = -\frac{40}{12} - \frac{40}{6}$$

$$= -\frac{20}{6} - \frac{40}{6} = \frac{-60}{6} = -10$$

$$\Rightarrow \boxed{\frac{\partial U}{\partial P_1} = D_1 = \frac{-10}{EI} \uparrow}$$

For Deflection @ P_2 substitute $P_1, P_2 = 0$

$$EI\frac{\partial U}{\partial P_2} = D_2 = \sum \int M.\frac{\partial M}{\partial P_2}\,dx$$

$$= \int_0^2 \frac{5}{4}x^2 + \int_2^4 \frac{5}{4}x^2 - \int_0^2 5\,x_2$$

$$= \frac{5}{4}\left|\frac{x^3}{3}\right|_0^2 + \frac{5}{4}\left|\frac{x^3}{3}\right|_2^4 - 5\left|\frac{x_2^2}{2}\right|_0^2$$

$$= \frac{5}{4}(2)^3 + \frac{5}{4}\left(\frac{4^3 - 2^3}{3}\right) - 5\frac{(2)^2}{2}$$

$$= \frac{40}{4} + \frac{5}{4} \times \frac{56}{3} - 5 \times \frac{4}{2} = \frac{280}{12}$$

$$= 23.33$$

$$\boxed{\frac{\partial U}{\partial P_2} = D_2 = \frac{23.33}{EI} \downarrow}$$

Solution – Assignment #2

Q.No 2

For Rotations do simillar procedure

Reactions

Taking moments @ B

$$4\overrightarrow{R_A} + M_1 + M_2 + 5\times 2 = 0 \Rightarrow R_A = \frac{-10 - M_1 - M_2}{4} \uparrow$$

$$R_B = 5 - R_A = 5 + \frac{10 + M_1 + M_2}{4} \Rightarrow R_B = \frac{30 + M_1 + M_2}{4} \uparrow$$

Range $2 \geq x \geq 0$

$$M = R_A \cdot x = \frac{-10 - M_1 - M_2}{4} \cdot x$$

$$\frac{\partial M}{\partial M_1} = -\frac{x}{4}$$

$$\frac{\partial M}{\partial M_2} = -\frac{x}{4}$$

$$M \frac{\partial M}{\partial M_1}\Bigg|_{M_1, M_2 = 0} = -\frac{10}{4} x \left(-\frac{x}{4}\right) = \frac{10x^2}{16}$$

Range $2 \leq x \leq 4$

$$\frac{\partial M}{\partial M_1} = -\frac{x}{4} + 1 = -\frac{(x-4)}{4}$$

$$\frac{\partial M}{\partial M_2} = -\frac{x}{4}$$

$$M = R_A x + M_1$$

$$M = \frac{-10 - M_1 - M_2}{4} \cdot x + M_1$$

$$M \frac{\partial M}{\partial M_1}\Bigg|_{M_1, M_2 = 0} = \frac{-10x}{4}\left(-\left(\frac{x-4}{4}\right)\right)$$

$$M \frac{\partial M}{\partial M_1}\Bigg|_{M_1, M_2 = 0} = \frac{10}{16}\left(x^2 - 4x\right)$$

$$M \frac{\partial M}{\partial M_2}\Bigg|_{M_1, M_2 = 0} = \frac{-10x}{4}\left(-\frac{x}{4}\right) = \frac{10x^2}{16}$$

Solution – Assignment #2

Q No. 2

Range $2 \geq x_2 \geq 0$

$$M = -5x_2 - M_2$$

$$\frac{\partial M}{\partial M_1} = 0$$

$$\frac{\partial M}{\partial M_2} = -1$$

$$M\frac{\partial M}{\partial M_1} = 0$$

$$M\frac{\partial M}{\partial M_2} = 5x_2 + M_2$$

$$M\frac{\partial M}{\partial M_2}\Big|_{M_1, M_2 = 0} = 5x_2$$

Apply Castigliano Theorum

$$EI\,\frac{\partial U}{\partial M_1} = \theta_1 = \sum \int M\frac{\partial M}{\partial M_1}$$

$$\theta_1 = \int_0^2 \frac{10x^2}{16} + \int_2^4 \frac{10}{16}(x^2 - 4x) + 0$$

$$= \frac{10}{16}\left|\frac{x^3}{3}\right|_0^2 + \frac{10}{16}\left|\frac{x^3}{3} - \frac{4x^2}{2}\right|_2^4$$

$$= \frac{10}{16}\frac{(2^3 - 0)}{3} + \frac{10}{16}\left[\frac{4^3 - 2^3}{3} - \frac{4}{2}(4^2 - 2^2)\right]$$

$$= 1.667 + -3.333 = -1.667$$

$$\Rightarrow \boxed{\frac{\partial U}{\partial M_1} = \theta_1 = \frac{-1.667}{EI}}$$

Solution - Assignment #2

For Rotation @ C

$$EI \frac{\partial U}{\partial M_2} = \sum \int M \frac{\partial M}{M_2}$$

$$\theta_2 = \int_0^2 \frac{10}{16} x^2 + \int_2^4 \frac{10x^2}{16} + \int_0^2 5 x_2 .$$

$$= \frac{10}{16} \left| \frac{x^3}{3} \right|_0^2 + \frac{10}{16} \left| \frac{x^3}{3} \right|_2^4 + 5 \left| \frac{x_2^2}{2} \right|_0^2$$

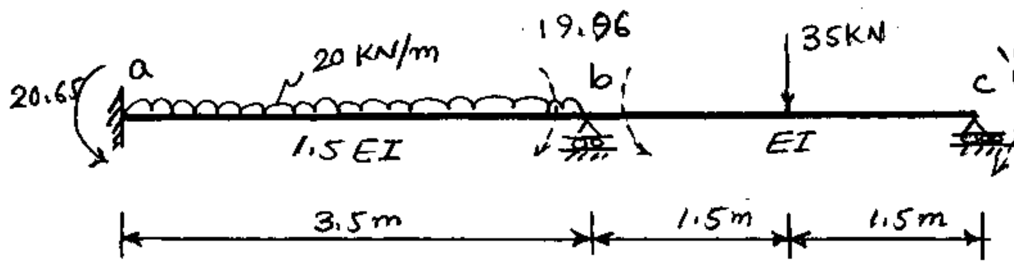
$$= \frac{10}{16} \frac{2^3}{3} + \frac{10}{16} \left[\frac{4^3 - 2^3}{3} \right] + 5 \left(\frac{2^2}{2} \right)$$

$$= 1.6667 + 11.6667 + 10 \qquad = 23.33$$

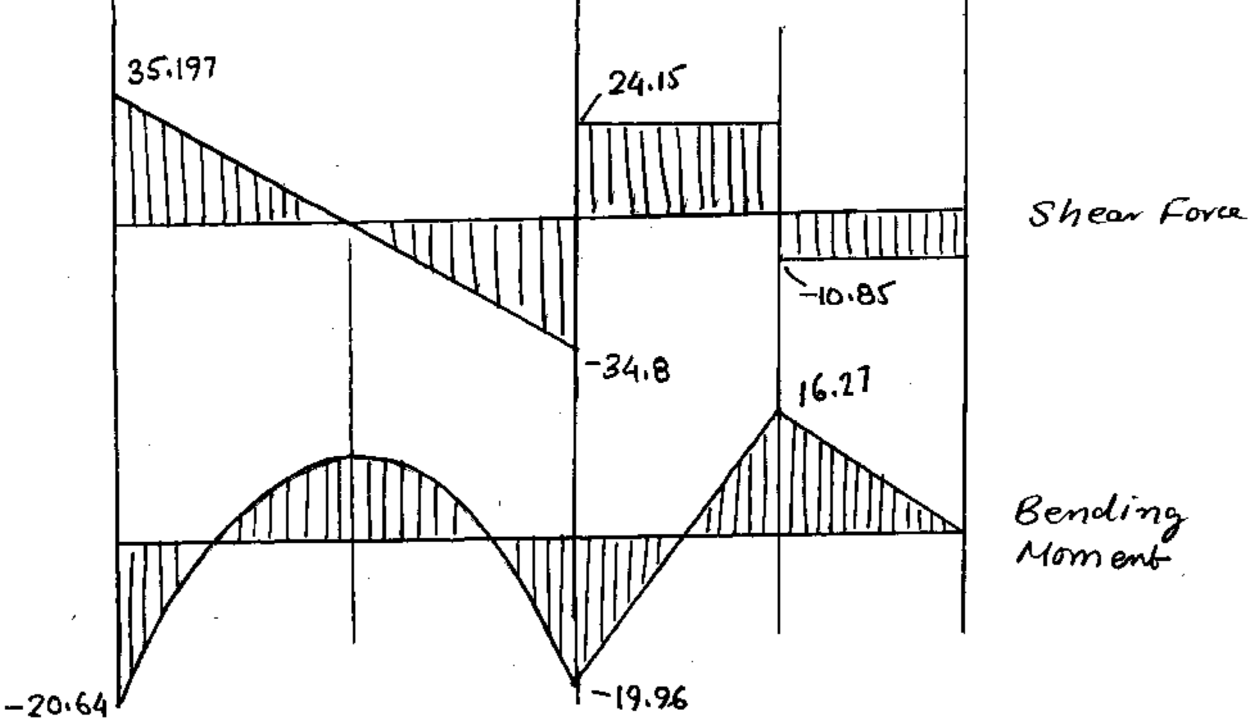
$$\Rightarrow \boxed{\frac{\partial U}{\partial M_2} - \theta_2 = \frac{23.33}{EI}}$$

Solution Assignment # 3

Q.No 1 Solve the Beam below using Moment Distribution Method. Generate Bending moment shear force diagram for the beam.



D.F	—	0.632	0.368	1
C.O	—	0.5	0.5	0.5
FEM	−20.42	+20.42	−13.13	+13.13
			−6.57	−13.13
	−0.23	−0.46	−0.26	
Final Moments	−20.65	+19.96	−19.96	0
R Load	+35	+35	+17.5	+17.5
R Moment	+0.197	−0.197	+6.65	−6.65
	35.197	+34.8	+24.15	+10.85



Assignment # 3

Q No. 1

Stiffness Factors

$$K_{ab} = \frac{1.5I}{3.5} = 0.4286\,I$$

$$K_{bc} = \frac{I}{3} = 0.3333\,I$$

$$K_{bc\,\text{modified}} = \frac{3}{4} \times \frac{I}{3} = 0.25\,I$$

Distribution Factors

$$D_{ba} = \frac{0.4286}{(0.4286 + 0.25)} = 0.632$$

$$D_{bc} = \frac{0.25}{(0.4286 + 0.25)} = 0.368$$

Fixed End Moments

$$FEM_{ab} = -\frac{w\ell^2}{12} = \frac{-20 \times (3.5)^2}{12} = -20.42 \text{ KN-m}$$

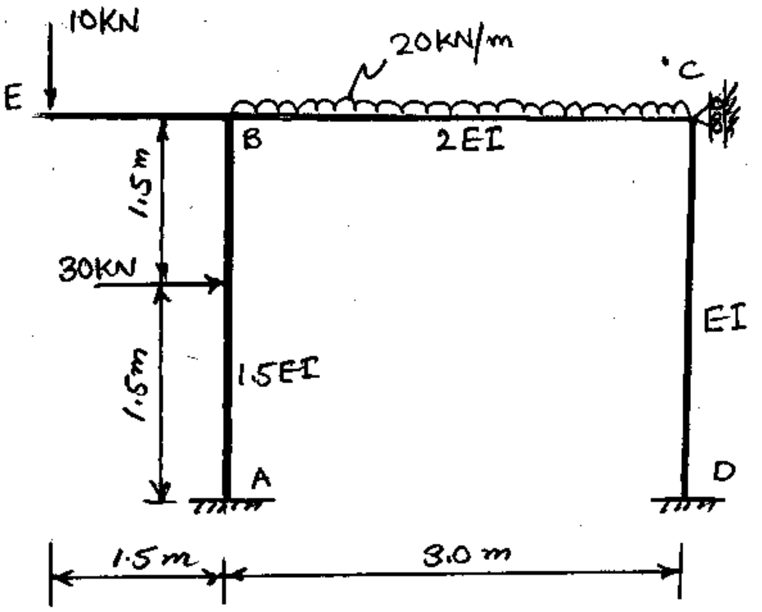
$$FEM_{ba} = +20.42 \text{ ''}$$

$$FEM_{bc} = -\frac{Pab^2}{\ell^2} = \frac{-35 \times 1.5 \times (1.5)^2}{(3)^2} = -13.13 \text{ KN-m}$$

$$FEM_{cb} = +13.13 \text{ KN-m}$$

Solution Assignment # 3

Q.No 2. Solve the frame shown below using moment distribution to find end moments. Draw Bending moment and shear force diagrams.



Stiffnesses & Relative Stiffnesses

$K_{BA} = K_{AB} = \frac{1.5I}{3} = 0.5I$

$K_{BC} = K_{CB} = \frac{2I}{3} = 0.667I$

$K_{CD} = K_{DC} = \frac{I}{3} = 0.333I$

Distribution Factors

$D_{BA} = \frac{0.5}{(0.5+0.667)} = 0.428$

$D_{BC} = \frac{0.667}{(0.5+0.667)} = 0.572$

$D_{CD} = \frac{0.333}{(0.333+0.667)} = 0.333$

$D_{CB} = \frac{0.667}{(0.333+0.667)} = 0.667$

Solution Assignment # 3

Fixed End moments

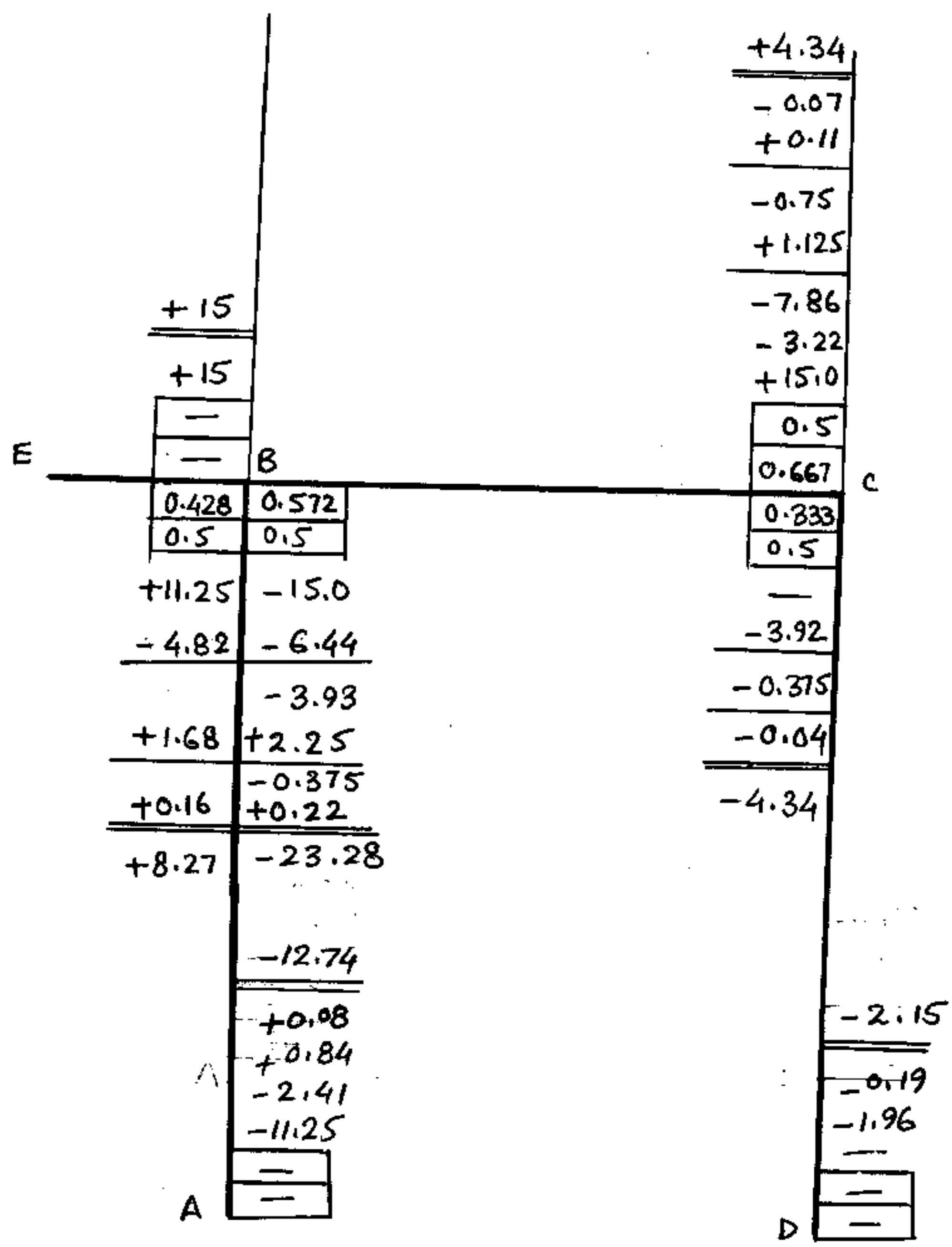
$$FEM_{AB} = -\frac{Pab^2}{l^2} = -\frac{30 \times 1.5 \times (1.5)^2}{(3)^2} = -11.25 \text{ KN-m}$$

$$FEM_{BA} = \qquad = +11.25 \text{ KN-m.}$$

$$FEM_{BC} = \frac{-wl^2}{12} = \frac{-20 \times (3)^2}{12} = -15 \text{ KN-m}$$

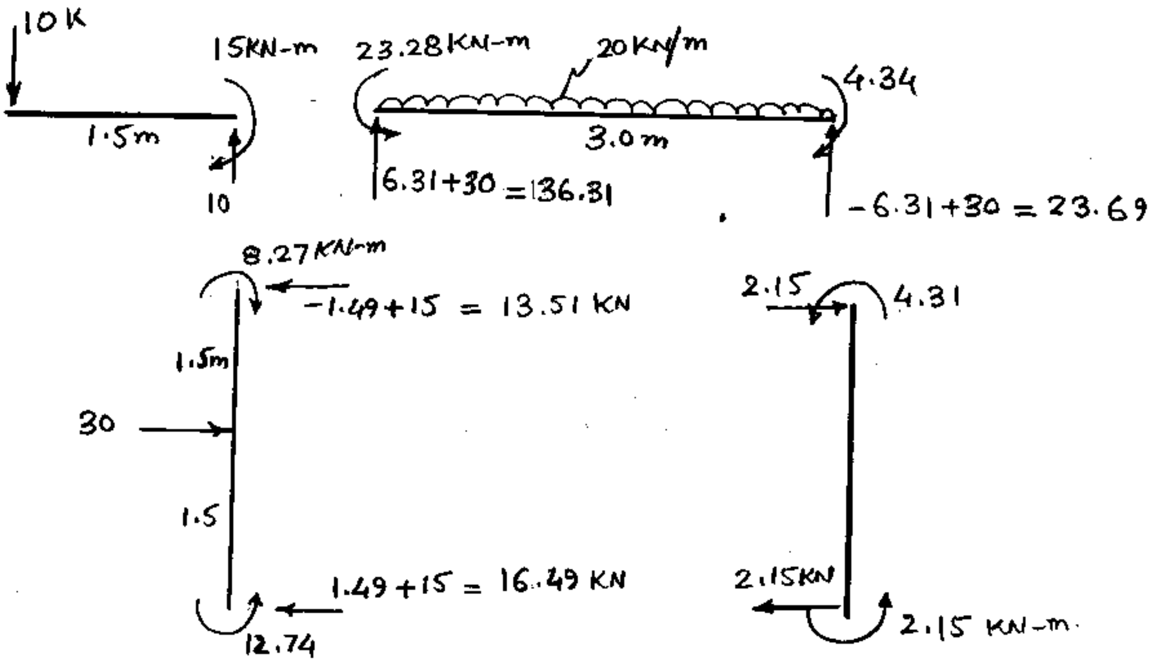
$$FEM_{CB} = +\frac{wl^2}{12} = \qquad = +15 \text{ KN-m}$$

$$FEM_{BE} = 10 \times 1.5 \qquad = +15 \text{ KN-m.}$$

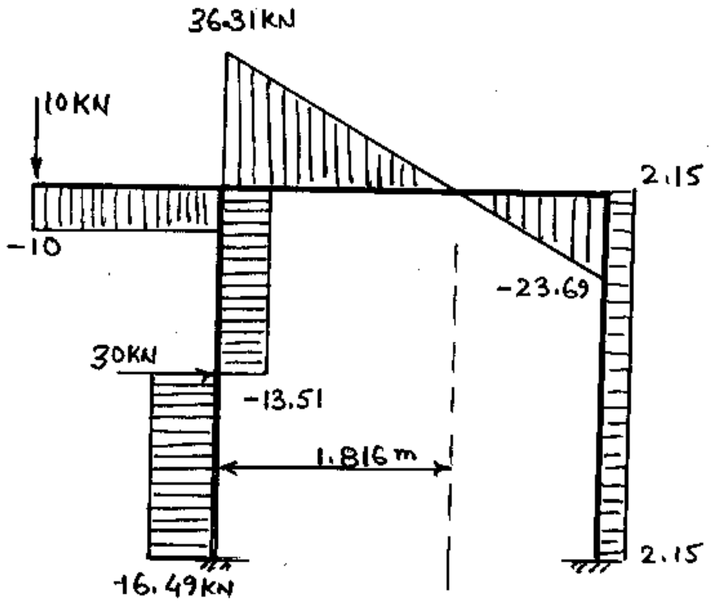


Solution Assignment # 3

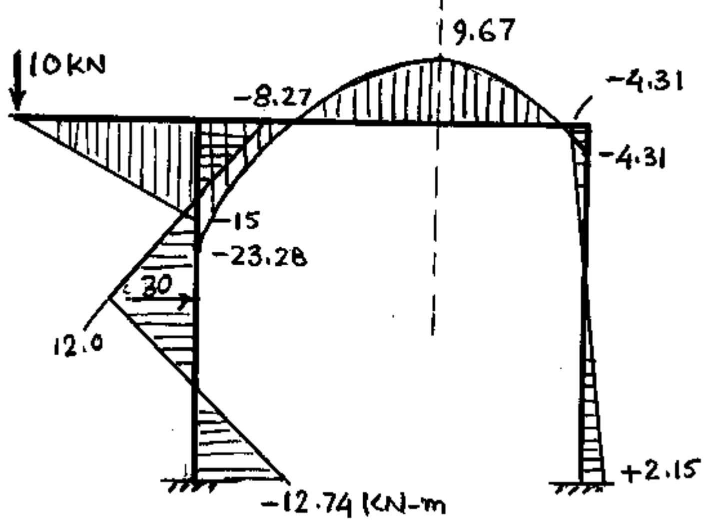
Q. No. 2



Free Body Diagrams



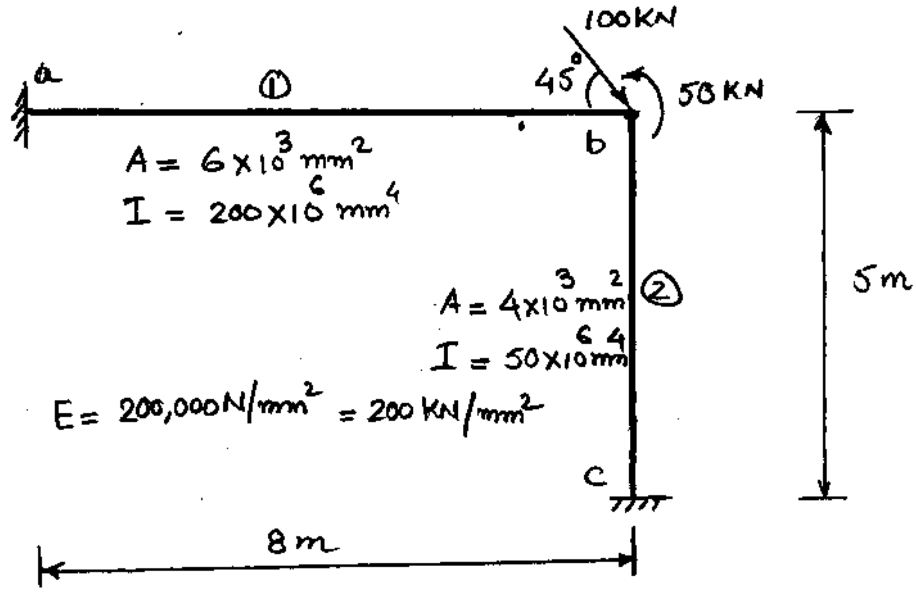
Shear Force



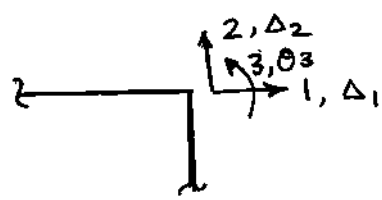
Bending Moment

Solution - Assignment # 4

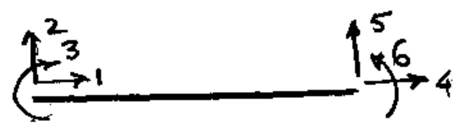
Q.1 Solve the Following Frame Structure using Matrix - Stiffness Method



Structure DOF



Member ab



$$K_{\substack{\text{Frame}\\ \text{Typical}}} = E\begin{bmatrix} \frac{A}{L} & 0 & 0 & -\frac{A}{L} & 0 & 0 \\ 0 & \frac{12I}{L^3} & \frac{6I}{L^2} & 0 & -\frac{12I}{L^3} & \frac{6I}{L^2} \\ 0 & \frac{6I}{L^2} & \frac{4I}{L} & 0 & -\frac{6I}{L^2} & \frac{2I}{L} \\ -\frac{A}{L} & 0 & 0 & \frac{A}{L} & 0 & 0 \\ 0 & -\frac{12I}{L^3} & -\frac{6I}{L^2} & 0 & \frac{12I}{L^3} & -\frac{6I}{L^2} \\ 0 & \frac{6I}{L^2} & \frac{2I}{L^2} & 0 & -\frac{6I}{L^2} & \frac{4I}{L} \end{bmatrix}$$

Solution - Assignment # 4

Member ab, 1

$$\frac{A}{L} = \frac{6000}{8000} = 0.75 \quad mm$$

$$\frac{12I}{L^3} = \frac{12 \times 200 \times 10^6}{(8000)^3} = 0.00469$$

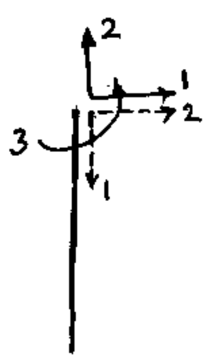
$$\frac{6I}{L^2} = \frac{6 \times 200 \times 10^6}{(8000)^2} = 18.75$$

$$\frac{4I}{L} = \frac{4 \times 200 \times 10^6}{(8000)} = 100{,}000$$

$$\frac{2I}{L} = \quad = 50{,}000$$

$$K_{ab} = K_{G_{ab}} = 200 \left[\begin{array}{ccc|ccc} & & & & & \\ & & & & & \\ & & & & & \\ \hline & & & 0.75 & 0 & 0 \\ & & & 0 & 0.00469 & -18.75 \\ & & & 0 & -18.75 & 100{,}000 \end{array}\right] \qquad D_{A_1} = \left\{\begin{array}{c} 0 \\ 0 \\ 0 \\ \hline 1 \\ 2 \\ 3 \end{array}\right\}$$

Member bc, 2



$$\frac{A}{L} = \frac{4000}{5000} = 0.8 \quad mm$$

$$\frac{12I}{L^3} = \frac{12 \times 50 \times 10^6}{(5000)^3} = 0.0048$$

$$\frac{6I}{L^2} = \frac{6 \times 50 \times 10^6}{(5000)^2} = 12.0$$

$$\frac{4I}{L} = \frac{4 \times 50 \times 10^6}{5000} = 40{,}000$$

$$\frac{2I}{L} = \quad = 20{,}000$$

$$K_{bc} = 200 \left[\begin{array}{ccc|ccc} 0.8 & 0 & 0 & & & \\ 0 & 0.0048 & 12 & & & \\ 0 & 12 & 40{,}000 & & & \\ \hline & & & & & \\ & & & & & \\ & & & & & \end{array}\right]$$

	x	Y	z
x'	0	-1	0
Y'	1	0	0
z'	0	0	1

$\underbrace{\qquad\qquad}_{[\gamma]}$

Solution - Assignment # 4

$$K_{G}^{bc} = K_{②G} = T^{T} K T$$

$$= 200 \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 0.0048 & 12 \\ 0 & 12 & 40{,}000 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$K_{2}G = 200 \begin{bmatrix} 0.0048 & 0 & 12 \\ 0 & 0.8 & 0 \\ 12 & 0 & 40{,}000 \end{bmatrix} \qquad DA_{②} = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$$

Structure Stiffness Matrix

$$K_G = 200 \begin{bmatrix} 0.75 + 0.0048 & 0 + 0 & 0 + 12 \\ 0 + 0 & 0.00469 + 0.8 & -18.75 + 0 \\ 0 + 12 & -18.75 + 0 & 100{,}000 + 40{,}000 \end{bmatrix} = 200 \begin{bmatrix} 0.7548 & 0 & 12 \\ 0 & 0.80469 & -18.75 \\ 12 & -18.75 & 140{,}000 \end{bmatrix}$$

Load Vector

$F_1 = 100 \cos 45 = 70.71$

$F_2 = -100 \cos 45 = -70.71$

$M_3 = 50\ \text{KN-m} = 50{,}000\ \text{KN-mm}$

Structure Equilibrium EQs

$$200 \begin{bmatrix} 0.7548 & 0 & 12 \\ 0 & 0.80469 & -18.75 \\ 12 & -18.75 & 140{,}000 \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} 70.71 \\ -70.71 \\ 50{,}000 \end{Bmatrix}$$

$K_G \qquad \Delta \qquad P$

$$\Rightarrow \boxed{\begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \ldots \end{Bmatrix} = \frac{1}{200} \begin{Bmatrix} 88.293 \\ -79.977 \\ 0.3388 \end{Bmatrix} = \begin{Bmatrix} 0.4414 \\ -0.3998 \\ 0.00169 \end{Bmatrix} \begin{matrix} \text{mm} \\ \text{mm} \\ \text{rad} \end{matrix}}$$

Soln

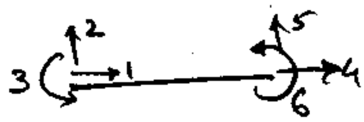
Solution – Assignment # 4

Member End Forces

Member ①, ab

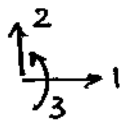
$$P = K\Delta + FEM$$

$$\left[\begin{array}{ccc|ccc}
 & & & -0.75 & 0 & 0 \\
 & & & 0 & -0.00469 & -18.75 \\
 & & & 0 & -18.75 & 50{,}000 \\ \hline
 & & & 0.75 & 0 & 0 \\
 & & & 0 & 0.00469 & -18.75 \\
 & & & 0 & -18.75 & 100{,}000
\end{array}\right]
\left\{\begin{array}{c} 0 \\ 0 \\ 0 \\ \hline 88.293 \\ -79.977 \\ 0.3388 \end{array}\right\} + 0 =
\left\{\begin{array}{c} -66.2 \\ 6.73 \\ 18439.6 \\ 66.22 \\ -6.73 \\ 35379.6 \end{array}\right\}$$



$$= \left\{\begin{array}{c} -66.2 \\ 6.73 \\ 18.44 \\ \hline 66.22 \\ -6.73 \\ 35.38 \end{array}\right\} \begin{array}{l} \\ \\ \text{KN-m} \\ \\ \\ \text{KN-m} \end{array}$$

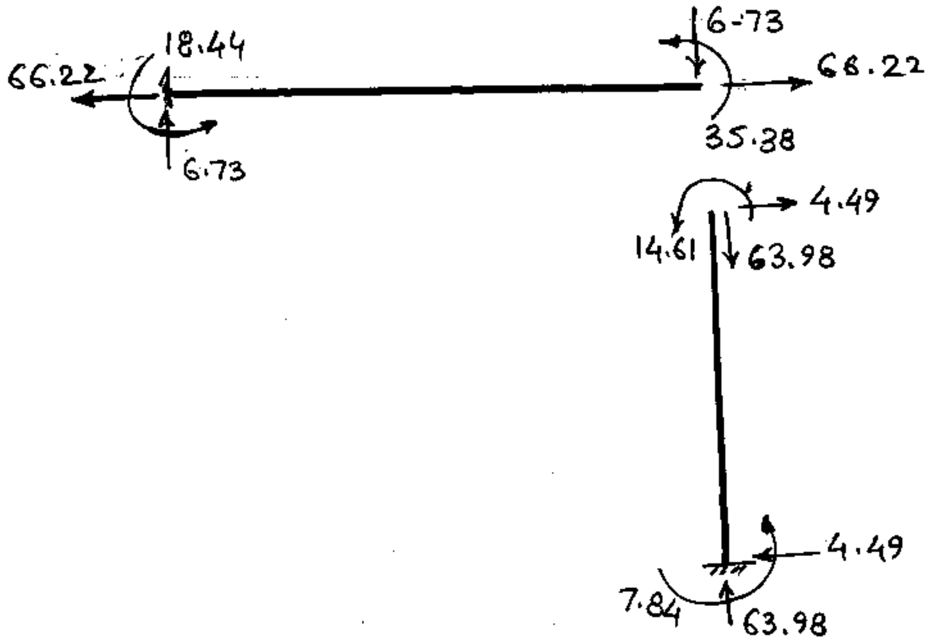
Member ② bc

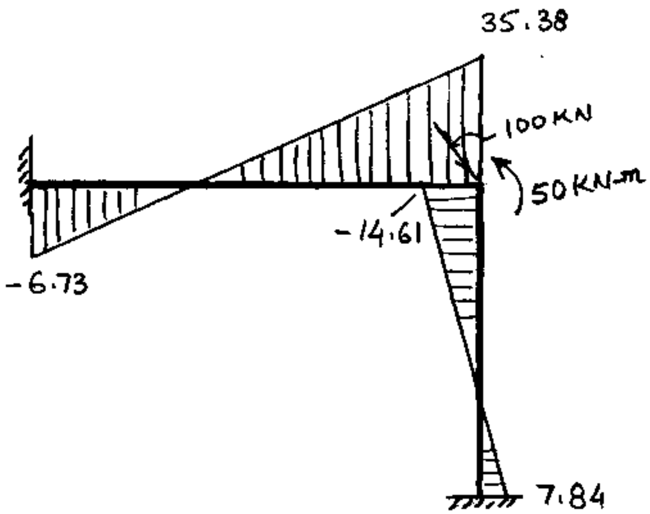


$$\left[\begin{array}{ccc|ccc}
0.8 & 0 & 0 & & & \\
0 & 0.0048 & 12 & & & \\
0 & 12 & 40{,}000 & & & \\ \hline
-0.8 & 0 & 0 & & & \\
0 & -0.0048 & -12 & & & \\
0 & 12 & 20{,}000 & & &
\end{array}\right]
\left\{\begin{array}{c} -79.977 \\ 88.293 \\ 0.3388 \\ \hline 0 \\ 0 \\ 0 \end{array}\right\} =
\left\{\begin{array}{c} 63.98 \\ 4.489 \\ 14611.5 \\ \hline -63.98 \\ -4.489 \\ 7835.5 \end{array}\right\} \begin{array}{l} \\ \\ \text{KN-mm} \\ \\ \\ \text{KN-mm} \end{array}$$

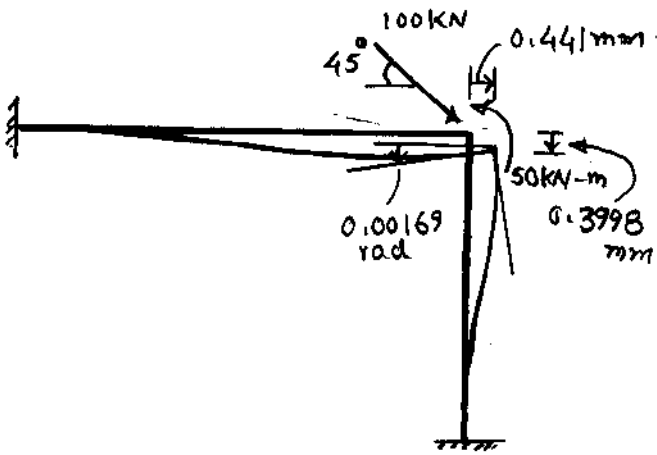
$$= \left\{\begin{array}{c} 63.98 \\ 4.489 \\ 14.61 \\ \hline -63.98 \\ -4.489 \\ 7.84 \end{array}\right\} \begin{array}{l} \text{KN} \\ \\ \text{KN-m} \\ \\ \\ \text{KN-m} \end{array}$$

Solution Assignment # 4





BENDING MOMENT



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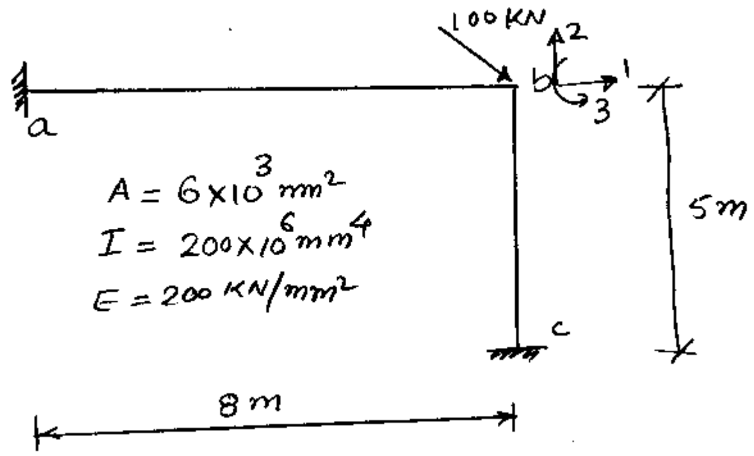
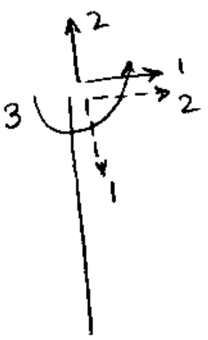
Solution – Assignment # 4

Case 2   If for member bc same section properties are taken as for member ab then rework the solution



Member 2 , bc



$$A/L = \frac{6000}{5000} = 1.2 \quad mm$$

$$\frac{12I}{L^3} = \frac{12\times200\times10^{6}}{(5000)^3} = 0.0192 \quad "$$

$$\frac{6I}{L^2} = \frac{6\times200\times10^{6}}{(5000)^2} = 48 \quad "$$

$$\frac{4I}{L} = \frac{4\times200\times10^{6}}{5000} = 160,000 \quad "$$

$$\frac{2I}{L} = \quad = 80,000 \quad "$$

$$K_{bc} = 200\left[\begin{array}{ccc|c} 1.2 & 0 & 0 & \\ 0 & 0.0192 & 48 & \\ 0 & 48 & 160,000 & \\ \hline & & & \\ & & & \end{array}\right] \qquad \gamma = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution - Assignment # 4

Case 2

$$K_{bc\,G} = T^T k T$$

$$= 200 \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.2 & 0 & 0 \\ 0 & 0.0192 & 48 \\ 0 & 48 & 160,000 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$K_2G = 200 \begin{bmatrix} 0.0192 & 0 & 48 \\ 0 & 1.2 & 0 \\ 48 & 0 & 160,000 \end{bmatrix} \qquad DA_2 = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$$

Assemble Structure Stiffness matrix

$$K_{G\,\text{Structure}} = 200 \begin{bmatrix} 0.75 + 0.0192 & 0 + 0 & 0 + 48 \\ 0 + 0 & 0.00469 + 1.2 & -18.75 + 0 \\ 0 + 48 & -18.75 + 0 & 100,000 + 160,000 \end{bmatrix}$$

$$K_{G\,\text{Structure}} = 200 \begin{bmatrix} 0.7692 & 0 & 48 \\ 0 & 1.20469 & -18.75 \\ 48 & -18.75 & 260,000 \end{bmatrix}$$

Solution Assignment #4

Case 2

Structure Equilibrium Eqns

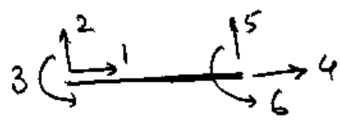
$$200\begin{bmatrix} 0.7692 & 0 & 48 \\ & 1.20469 & -18.75 \\ \text{Sym} & & 260{,}000 \end{bmatrix}\begin{Bmatrix}\Delta_1\\ \Delta_2\\ \theta_3\end{Bmatrix} = \begin{Bmatrix}70.71\\ -70.71\\ 50{,}000\end{Bmatrix}$$

$$\underset{K_G}{}\qquad\qquad \Delta \qquad\qquad P$$

$$\Rightarrow \begin{Bmatrix}\Delta_1\\ \Delta_2\\ \theta_3\end{Bmatrix} = \frac{1}{200}\begin{Bmatrix}81.2\\ -56.0\\ 0.174\end{Bmatrix} = \begin{Bmatrix}0.406\\ -0.28\\ 0.00087\end{Bmatrix} \quad \text{Answer}$$

Member End Forces

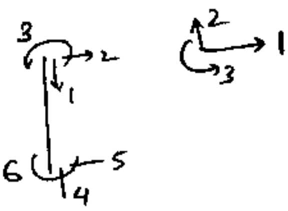
Member 1, ab



$$P = K\Delta + FEM$$

$$\left[\begin{array}{c|ccc} & -0.75 & 0 & 0 \\ & 0 & -0.00469 & 18.75 \\ & 0 & -18.75 & 50{,}000 \\ \hline & 0.75 & 0 & 0 \\ & 0 & 0.00469 & -18.75 \\ & 0 & -18.75 & 100{,}000 \end{array}\right]\begin{Bmatrix}0\\0\\0\\ \hline 81.2\\ -56.0\\ 0.174\end{Bmatrix} = \begin{Bmatrix}-60.9\\ 3.5\\ 9750\\ \hline 60.9\\ -3.5\\ 18450\end{Bmatrix}\begin{matrix}\text{KN}\\ \text{KN}\\ \text{KN-mm}\\ \text{KN}\\ \text{KN}\\ \text{KN-mm}\end{matrix}$$

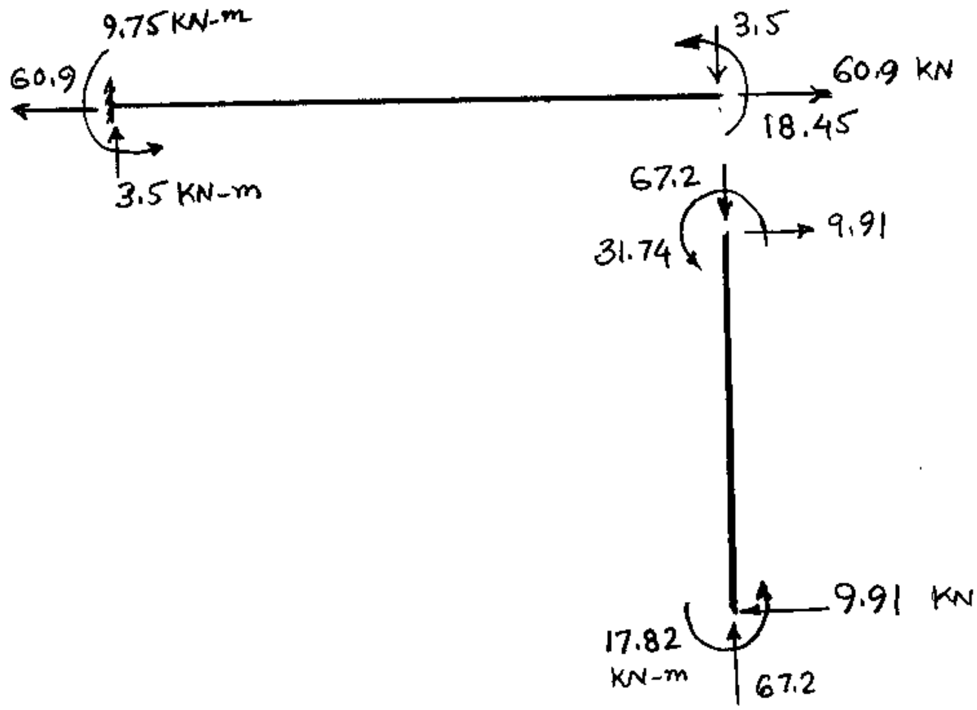
Member bc

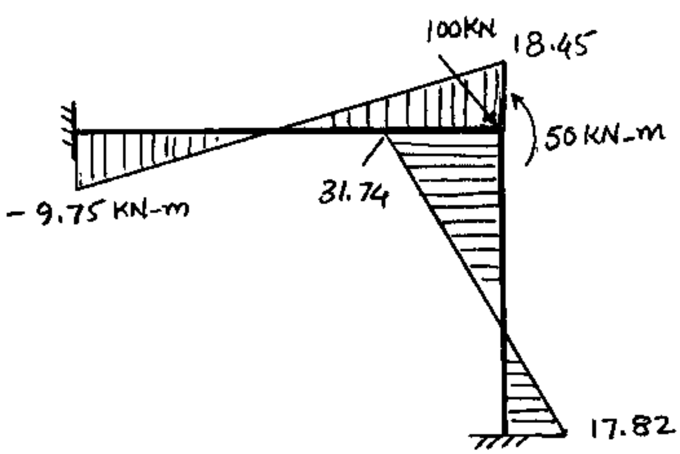


$$\left[\begin{array}{ccc|c} 1.2 & 0 & 0 & \\ 0 & 0.0192 & 48 & \\ 0 & 48 & 160{,}000 & \\ \hline -1.2 & 0 & 0 & \\ 0 & -0.0192 & -48 & \\ 0 & 48 & 80{,}000 & \end{array}\right]\begin{Bmatrix}56.0\\ 81.2\\ 0.174\\ \hline 0\\0\\0\end{Bmatrix} = \begin{Bmatrix}67.2\\ 9.91\\ 31{,}737\\ \hline -67.2\\ -9.91\\ 17817.6\end{Bmatrix}\begin{matrix}\text{KN}\\ \text{KN}\\ \text{KN-mm}\\ \text{KN}\\ \text{KN}\\ \text{KN-mm}\end{matrix}$$

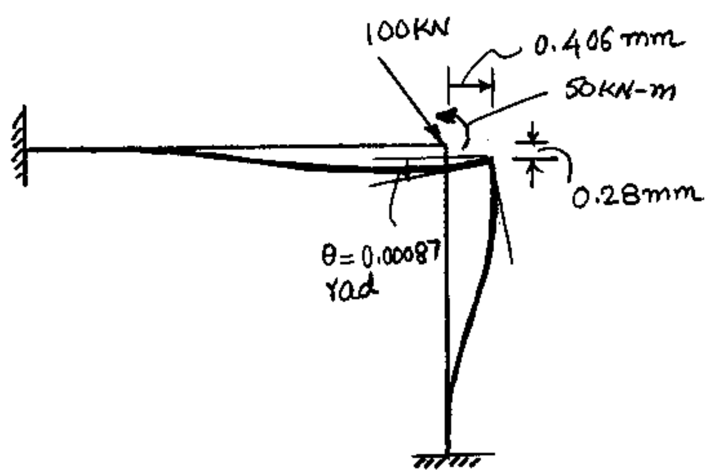
Solution - Assignment # 4

CASE 2





BENDING MOMENT



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CASE-2