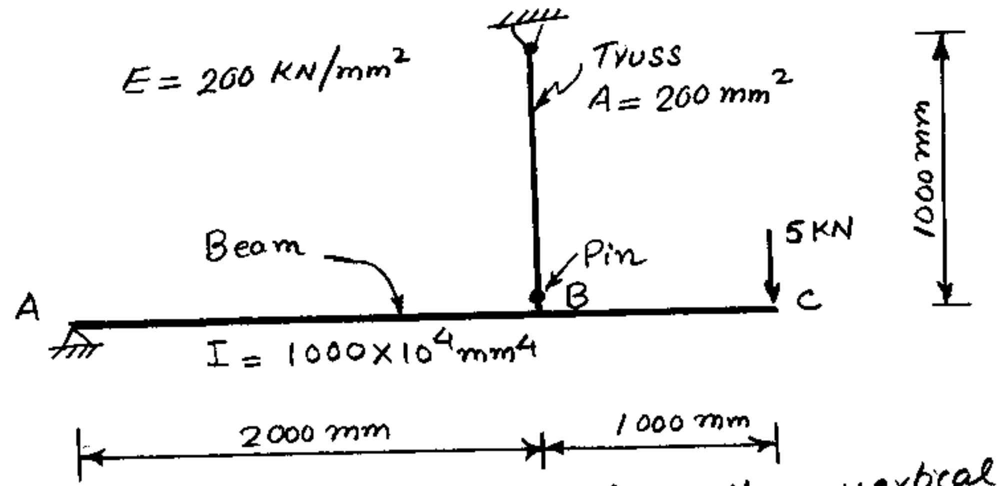
FINAL EXAM – CE-5111
Advanced Structural Analysis

Time Allowed : 2:30 hrs
Closed Book
Closed Notes

Q. No. 1

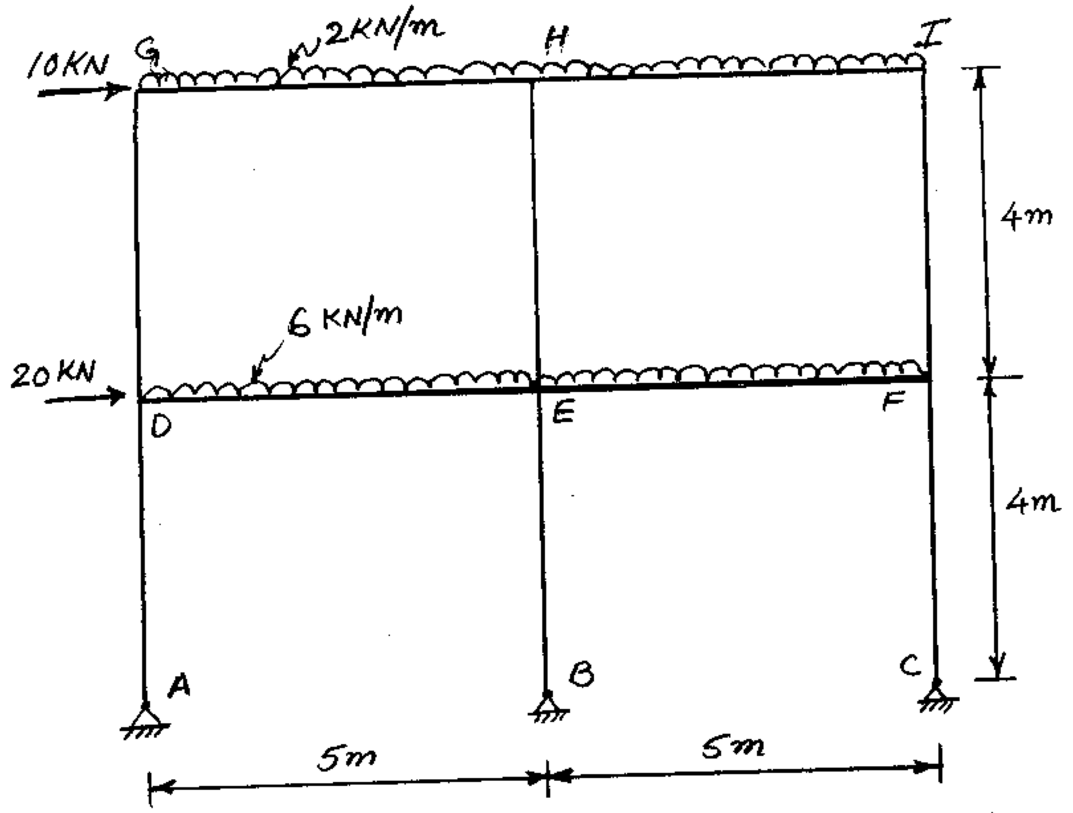
(30 Marks)



For the structure shown above, find the vertical deflection at Pt "C" using method of virtual work.

Q.No 2

(40 Marks)



Analyze the frame structure shown above using Approximate Analysis Technique. Determine and Plot the approximate moments along Column Line "ADG" only.

(P.T.O)

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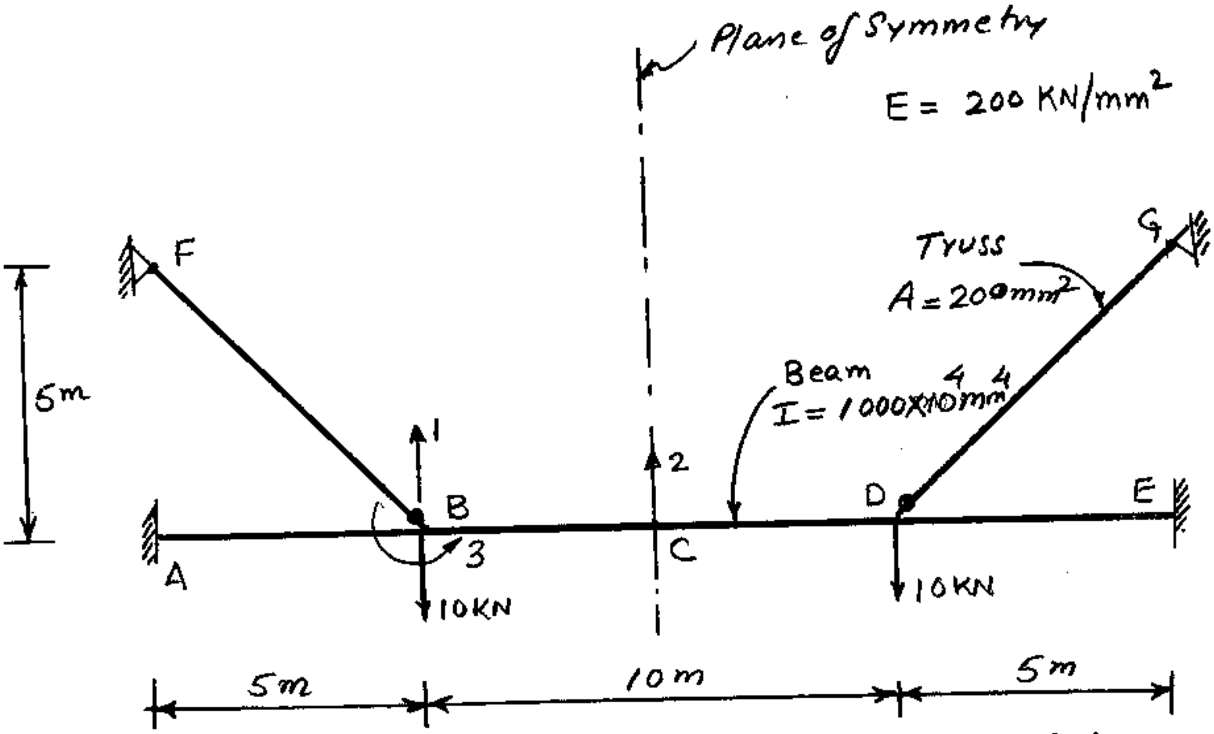
Advanced Structural Analysis

Q. No. 3 (30 Marks)

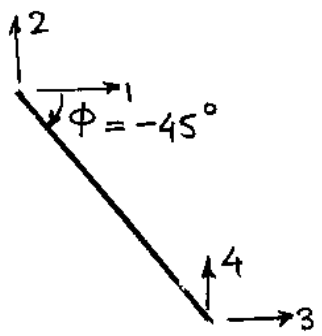


Analyze the structure shown above using Matrix Analysis/Stiffness Method. Neglect the axial deformations in the beam members.

a) Find the structural degrees of freedom indicated on the figure

b) Find the axial force in Truss member FB

The stiffness matrix of Truss member FB in global coordinates is given below:



$$K_{\substack{FB \\ Global}} = \frac{EA}{L}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & s^2 & sc & s^2 \end{bmatrix}$$

$s = \sin\phi$, $c = \cos\phi$

Hint: Utilize the symmetry of the structure and loading to simplify the problem.

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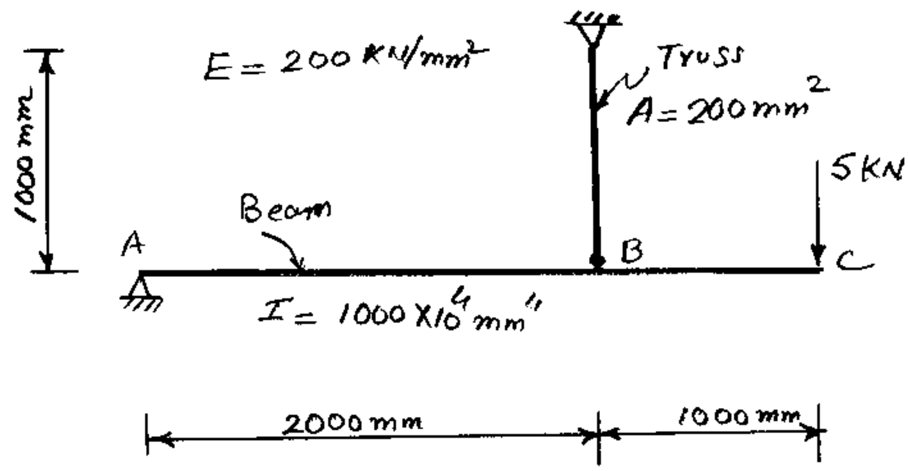
Q No. 1



For the structure shown above, find the deflection at pt C using method of virtual work.

Determine Reactions

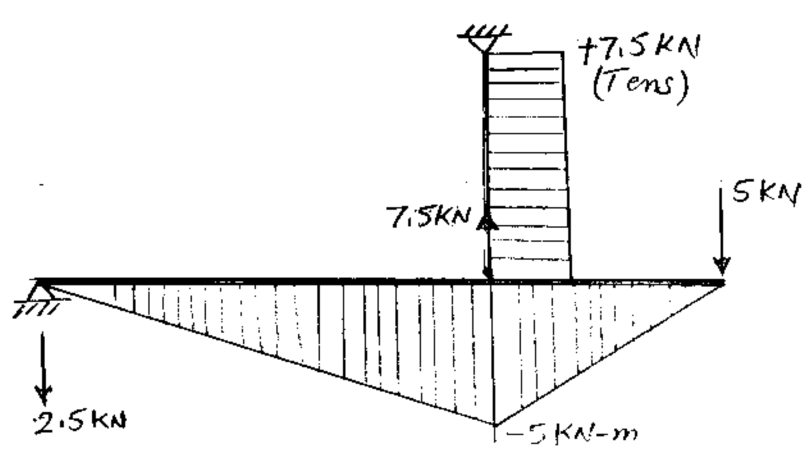
Structure is statically determinate.

Taking moments @ A

$2000\, R_B - 5 \times 3000 = 0 \Rightarrow R_B = \frac{5 \times 3000}{2000} = 7.5\ \text{KN} \uparrow$

Truss member has tension $= 7.5$ KN. (Tension)

$R_A = 5 - 7.5 \Rightarrow R_A = -2.5\ \text{KN} \downarrow$



Bending Moment & Axial Force Diagram for Structure

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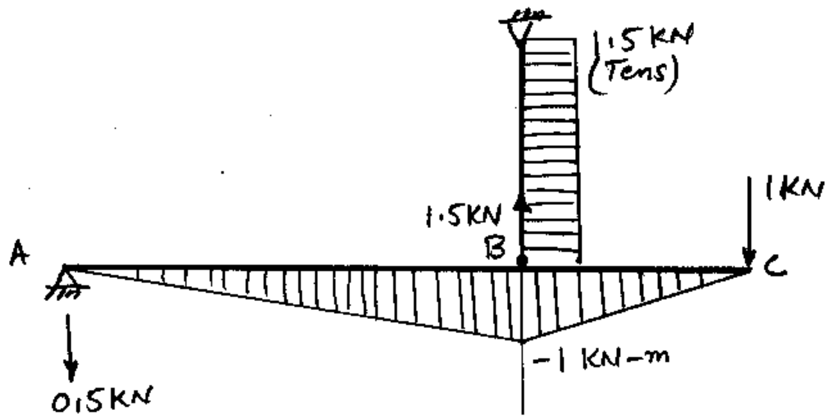
QNo 1.

For Deflection @ C apply unit load at pt C
The bending moment and axial Forces are as shown
below for unit/dummy load.



Apply Principle of
Virtual Work

$$1.\ \Delta_c = \sum \int \frac{M m}{EI} dx + \sum \frac{F f L}{AE}$$

$$= \frac{1}{EI}\left[\int_0^2 \left(-\frac{5}{2}x\right)\cdot\left(-\frac{1}{2}x\right) + \int_0^1 (-5x)(-x)\right]$$

$$+ \frac{1}{AE}\ (7.5 \times 1.5 \times 1)$$

$$= \frac{1}{EI}\left[\int_0^2 \frac{5}{4}x^2 + \int_0^1 5x^2\right] + \frac{11.25}{AE}$$

$$= \frac{1}{EI}\left[\frac{5}{4}\left|\frac{x^3}{3}\right|_0^2 + 5\left|\frac{x^3}{3}\right|_0^1\right] + \frac{11.25}{AE}$$

$$= \frac{1}{EI}\left[\frac{5}{4} \times \frac{(2)^3}{3} + \frac{5}{3}\right] + \frac{11.25}{AE}$$

$$= \frac{1}{EI} \times 5 + \frac{11.25}{AE} \quad (KN\text{-}m)$$

$A = 200\ mm^2 = 2 \times 10^{-4} m^2$
$I = 1000 \times 10^4\ mm^4$
$= 1 \times 10^{-5} m^4$
$E = 200\ KN/mm^2$
$= 200 \times 10^6\ KN/m^2$

$$= \frac{5}{200 \times 10^6 \times 1 \times 10^{-5}} + \frac{11.25}{2 \times 10^{-4} \times 200 \times 10^6}$$

$$\Rightarrow \Delta_c = 2.5 \times 10^{-3} + 0.28125 \times 10^{-3} = 2.7813 \times 10^{-3}\ m = 2.78\ mm \downarrow$$

Ans

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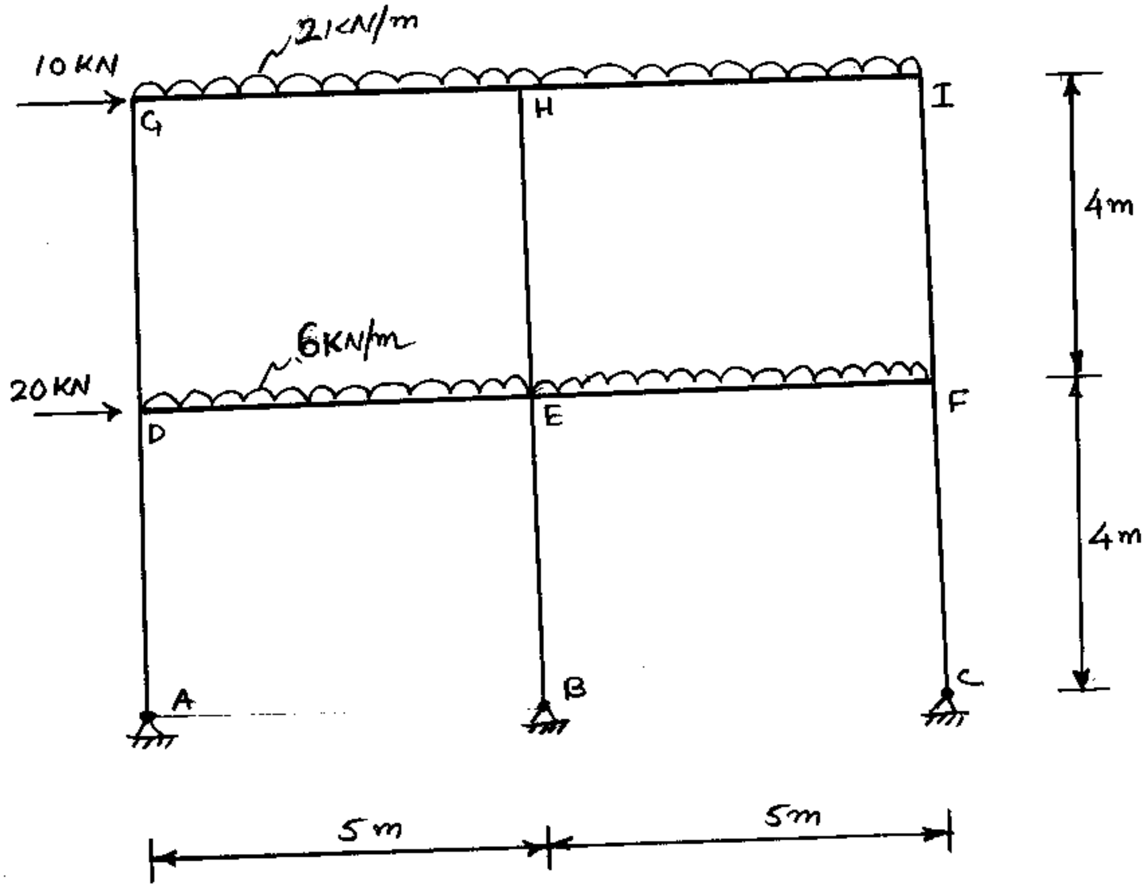
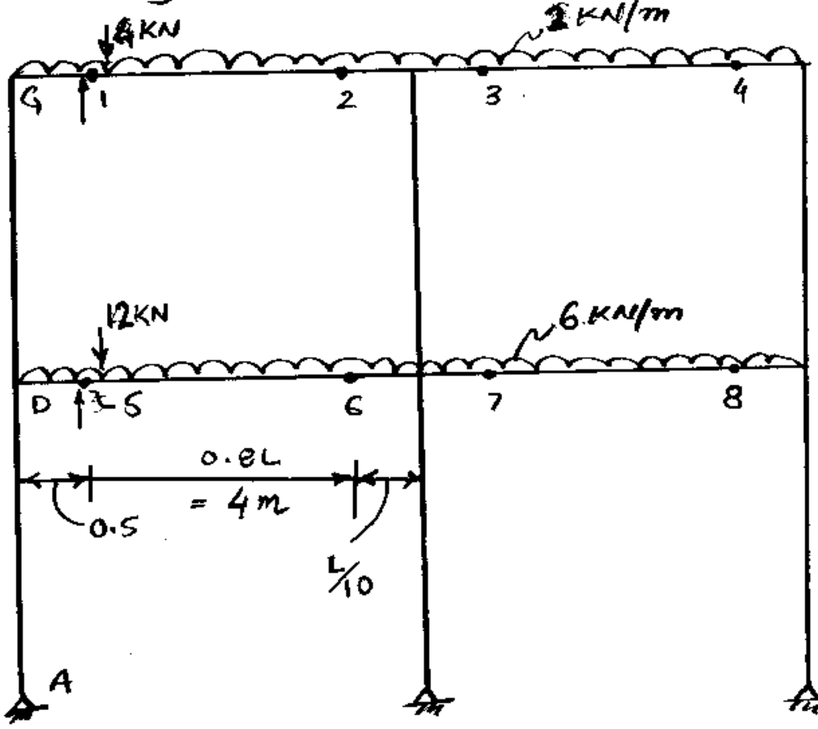
QNo.3



Analyze the Frame shown above using approximate Analysis technique. Determine and plot the moments along Column line ADG.

Solution

First carry out analysis for gravity load



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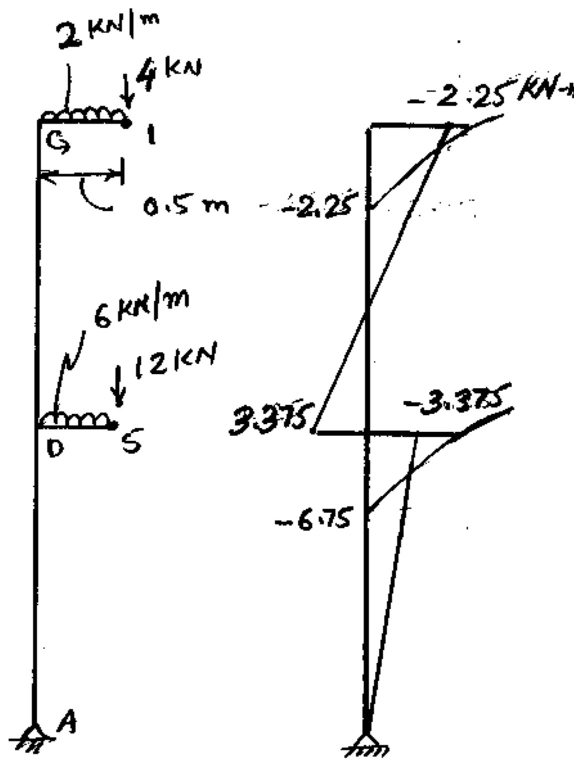
QNo.3

Moment @ G = $4 \times 0.5 + 2 \times 0.5 \times \frac{0.5}{2}$

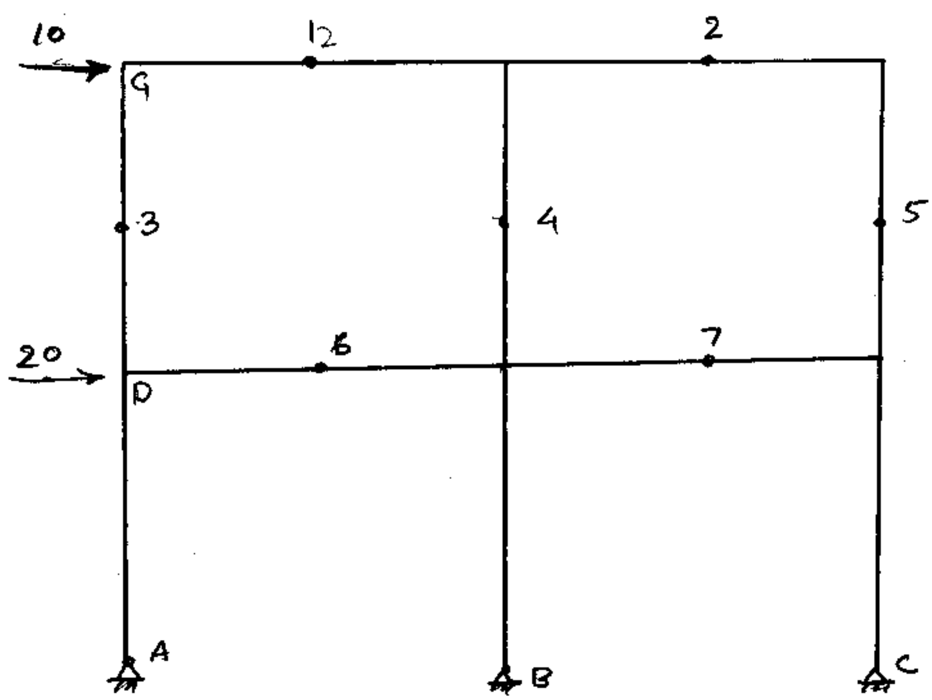
= 2.25 KN - m

Moment @ D = $12 \times 0.5 + 6 \times \frac{(0.5)^2}{2}$

= 6.75 KN

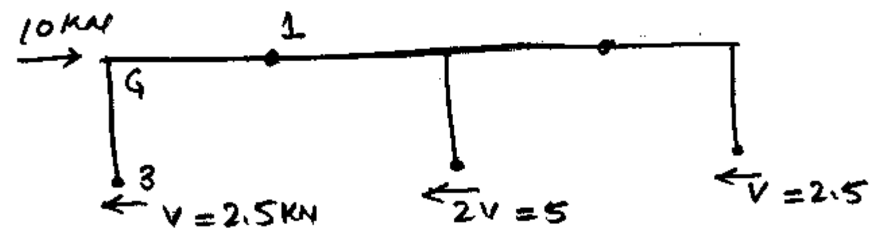


Carrying out Lateral Load Analysis



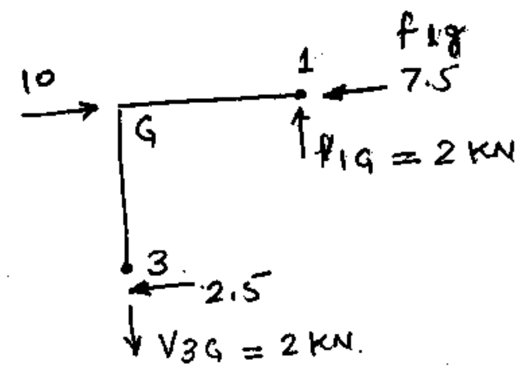
Q No. 3

Consider upper story



$4V = 10 \Rightarrow V = \frac{10}{4} = 2.5$ KN

$f_{1G} = 10 - 2.5 = 7.5$

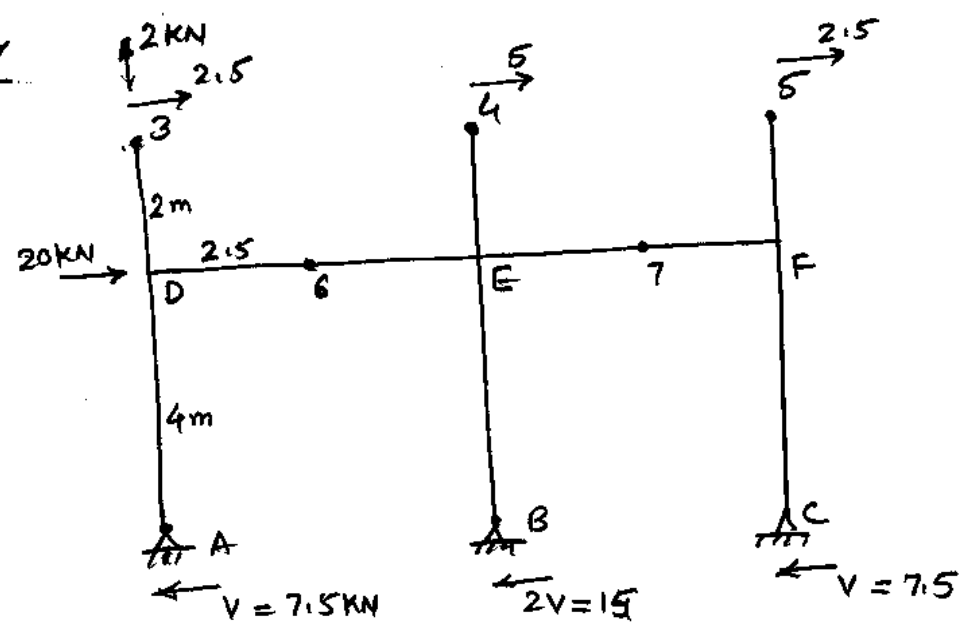


Taking moments at G

$V_{1G} \times 2.5 - 2.5 \times 2 = 0$

$V_{1G} = \frac{2.5 \times 2}{2.5} = 2$ KN ↑

Consider Lower storey

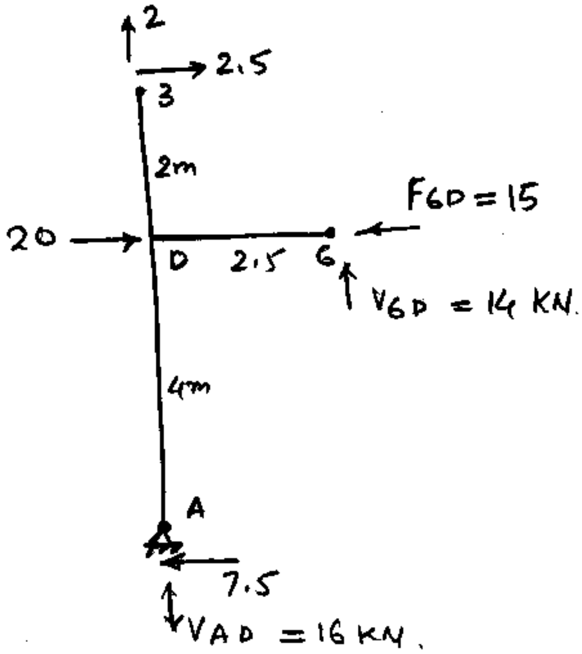


$4V = 10 + 20 = 30$

$V = \frac{30}{4} = 7.5$ KN

QNo 3

Consider Bent AD G 3



$F_{6D} + 7.5 - 2.5 - 20 = 0$

$F_{6D} = 15$

Moment @ D

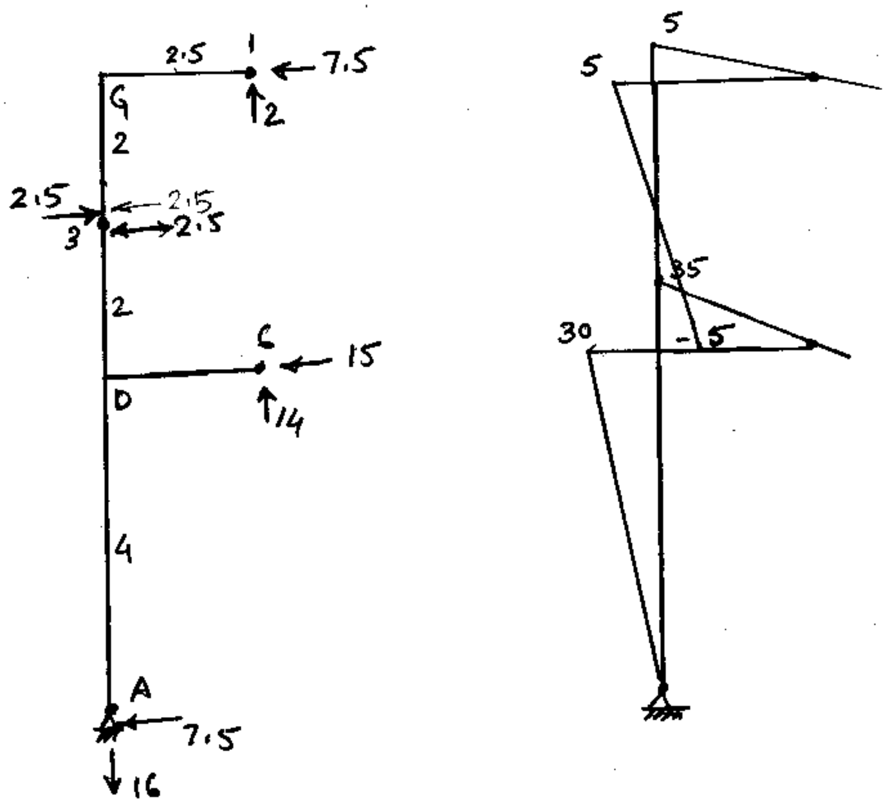
$V_{6D} \times 2.5 - 2.5 \times 2 - 7.5 \times 4 = 0$

$\Rightarrow V_{6D} = \frac{35}{2.5} = 14 KN \uparrow$

$V_{AD} - 14 - 2 = 0$

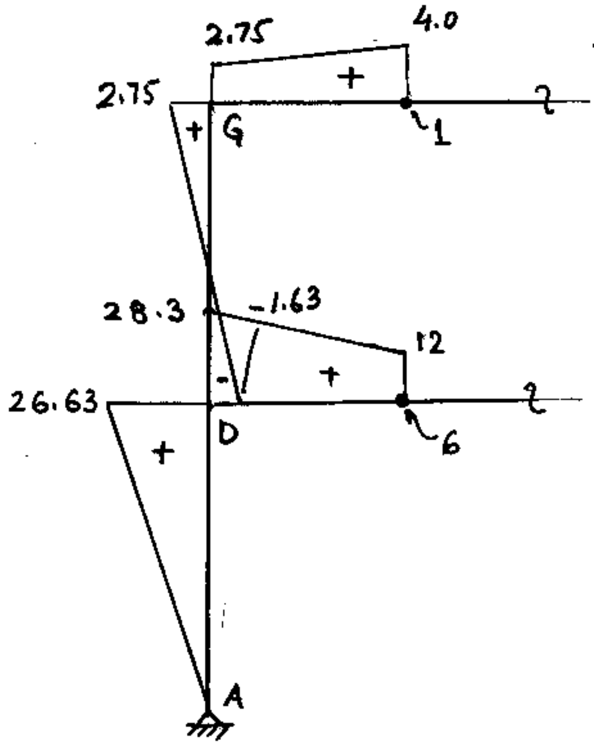
$\Rightarrow V_{AD} = 16 KN$

Force moment summary for column line ADG

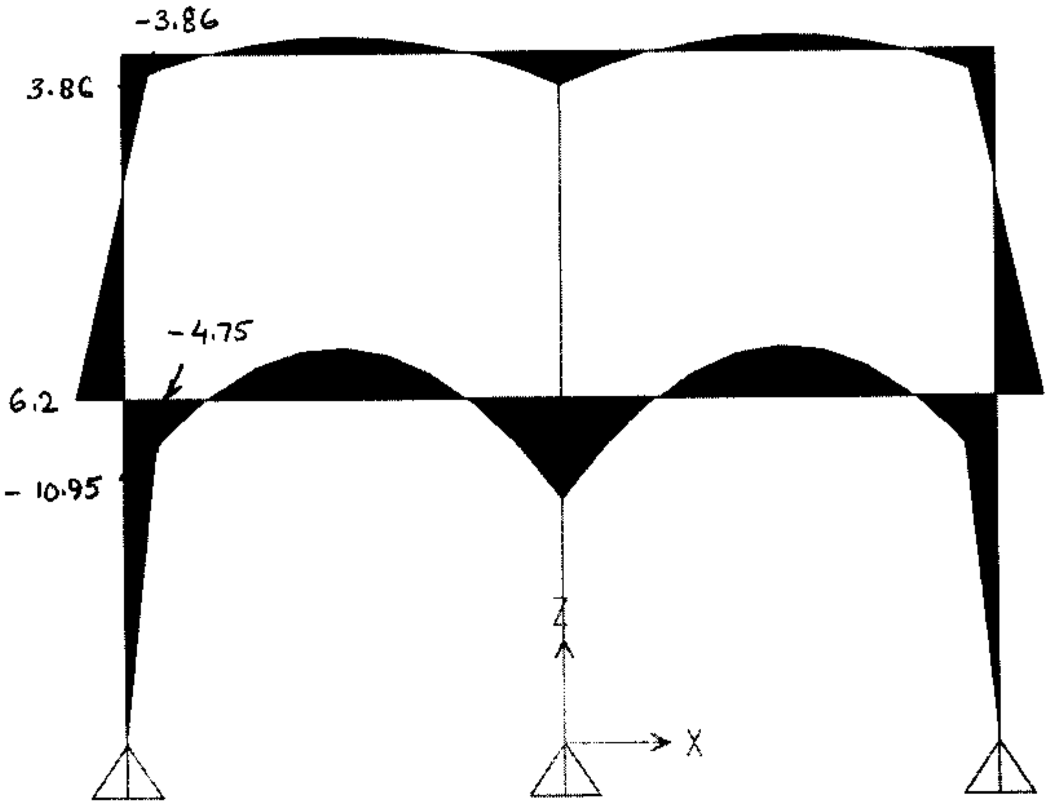


Q No 3

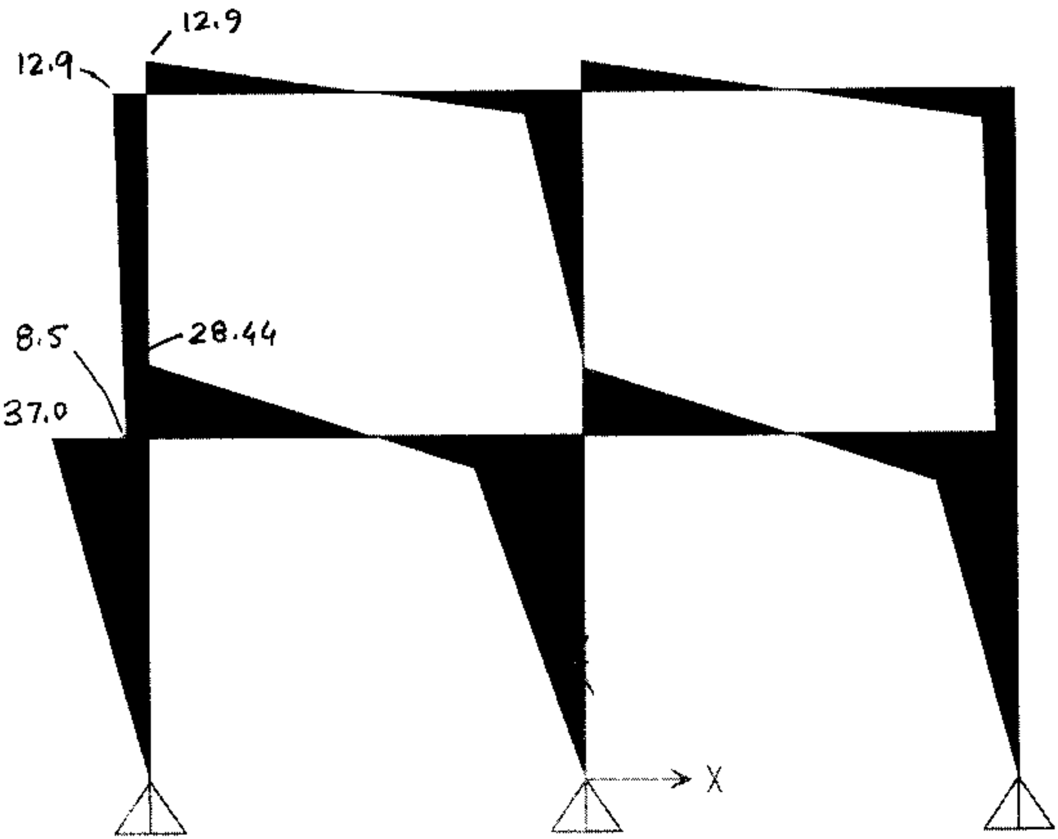
Superimposing the results for gravity & lateral we get the final moments in column line.



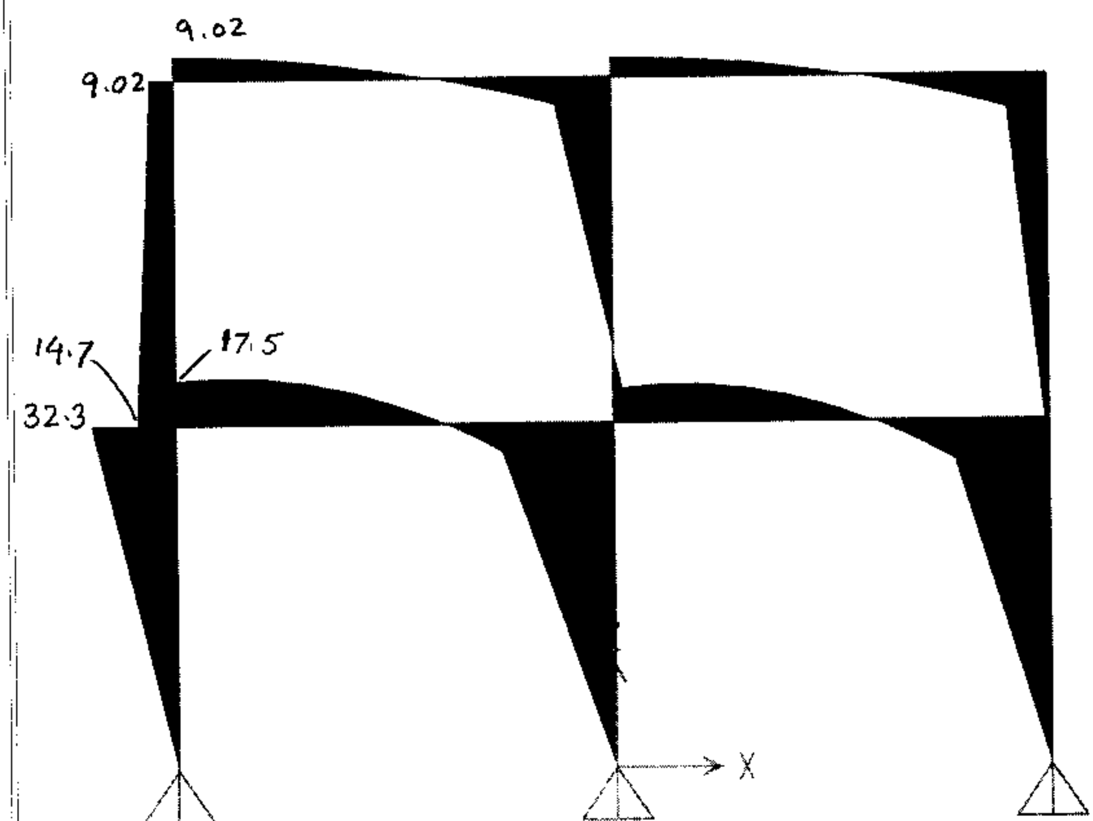
FINAL COMBINED BENDING MOMENT DIAGRAM OF COLUMN ADG



SAP ANALYSIS OF GRAVITY LOAD



SAP ANALYSIS OF LATERAL LOAD



SAP ANALYSIS OF COMBINED GRAVITY & LATERAL LOADS

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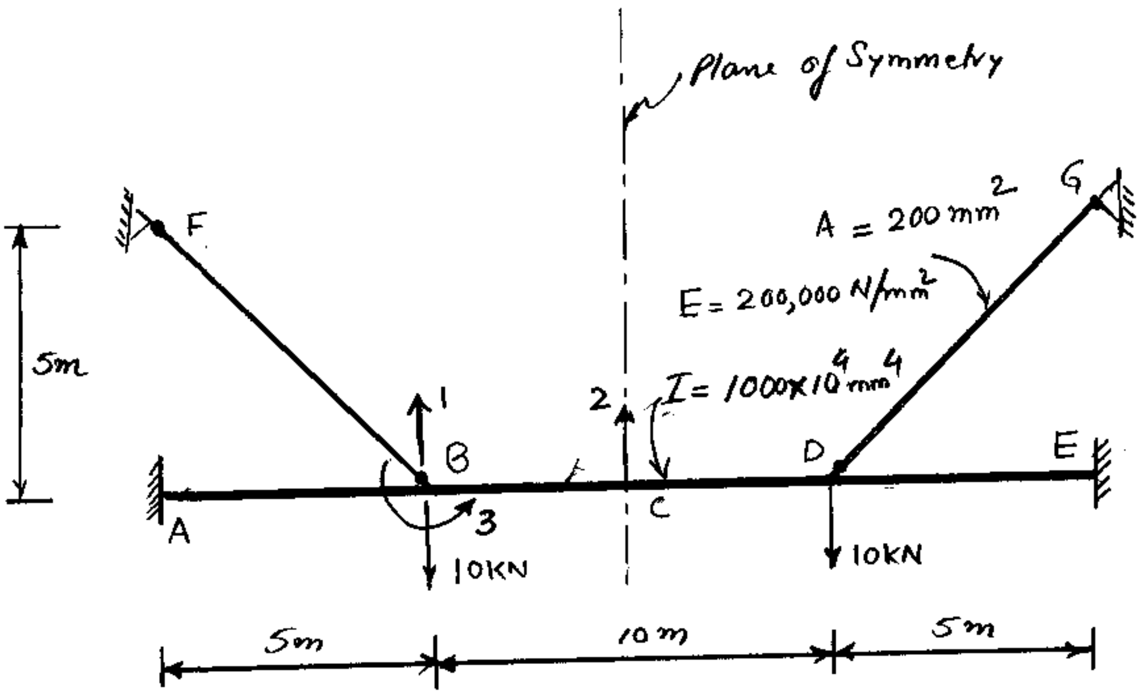
QNo. 3



Analyze the structure shown above using the matrix analysis stiffness method. Find the displacements at pts B & C. The structural degrees of freedom are indicated on the structure. Assume that the Frame member ABCDE is axially rigid.

a) Find the displacements at pts B & C

b) Find the axial Force in inclined truss elements FB & GD

Hint: Utilize Symmetry of structure and loading

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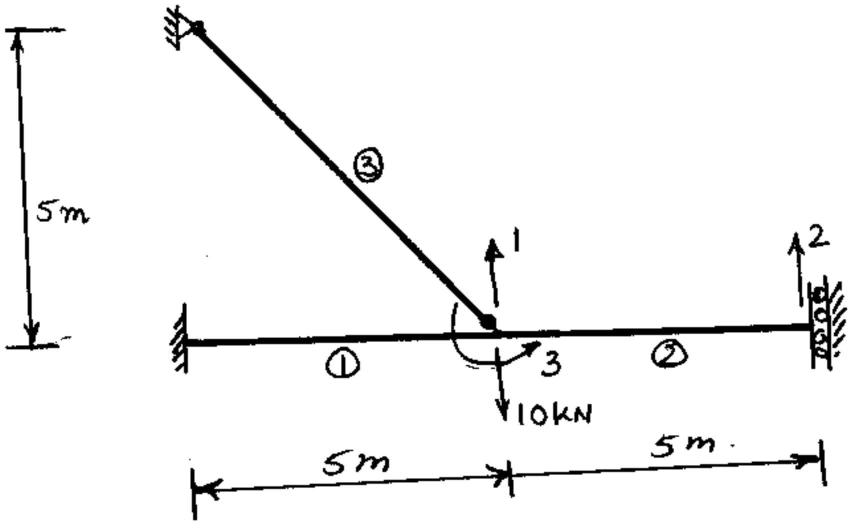
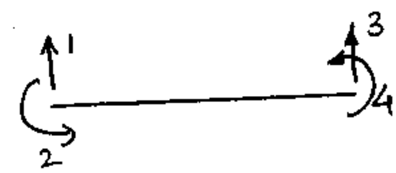
Q No 3

Soln Utilizing symmetry the problem can be recast as follows



Member ①



$$K = E \begin{bmatrix} \frac{12I}{L^3} & \frac{6I}{L^2} & -\frac{12I}{L^3} & \frac{6I}{L^2} \\ \frac{6I}{L^2} & \frac{4I}{L} & -\frac{6I}{L^2} & \frac{2I}{L} \\ -\frac{12I}{L^3} & -\frac{6I}{L^2} & \frac{12I}{L^3} & -\frac{6I}{L^2} \\ \frac{6I}{L^2} & \frac{2I}{L} & -\frac{6I}{L^2} & \frac{4I}{L} \end{bmatrix}$$

$$\frac{12I}{L^3} = \frac{12 \times 1000 \times 10^4}{(5000)^3} = 0.00096 \text{ mm}, \quad \frac{4I}{L} = \frac{4 \times 1000 \times 10^4}{5000} = 8000$$

$$\frac{6I}{L^2} = \frac{6 \times 1000 \times 10^4}{(5000)^2} = 2.4 \qquad \frac{2I}{L} = 4000$$

$$K_{①} = K_{②} = 200 \begin{array}{c} \begin{array}{cccc} 1 & 3 & 2' & \end{array} \\ \begin{bmatrix} 0.00096 & 2.4 & -0.00096 & 2.4 \\ 2.4 & 8000 & -2.4 & 4000 \\ -0.00096 & -2.4 & 0.00096 & -2.4 \\ 2.4 & 4000 & -2.4 & 8000 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 3 \\ 2 \\ \\ \end{array} \quad DA_{①} = \begin{Bmatrix} 0 \\ 0 \\ \hline 1 \\ 3 \end{Bmatrix}$$

$$DA_{②} = \begin{Bmatrix} 1 \\ 3 \\ \hline 2 \\ 0 \end{Bmatrix}$$

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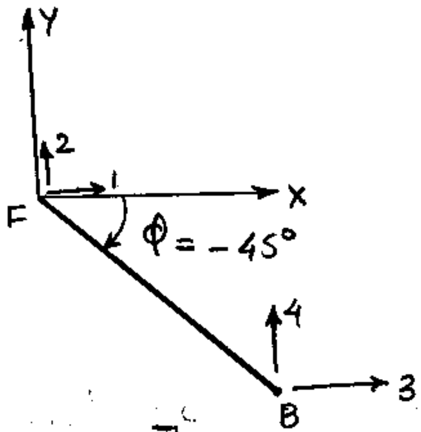
QNo 3

Soln

Truss member 3



$$L = \sqrt{5^2 + 5^2} = 5\sqrt{2}\ m$$

$$= 7.071 m = 7071\ mm$$

$$K3 = \frac{EA}{L} \begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix} \qquad DA3 = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix}$$

$c = \cos\phi$, $c^2 = \cos^2(-45) = 0.5$

$s = \sin\phi$ $\quad s^2 = \sin^2(-45) = 0.5$

$sc = \cos(-45)\sin(-45) = -0.5$

$$\frac{EA}{L} = E \times \frac{200}{7071} = 0.02828\ E$$

$$K3 = 0.01414\ E \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

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QN03

Structural Stiffness Matrix

$$K_{G\,struc} = 200 \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.00096 + 0.00096 + 0.01414 & -0.00096 & -2.4 + 2.4 \\ -0.00096 & 0.00096 & -2.4 \\ 2.4 & -2.4 & 8000 + 8000 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \end{array}$$

$$K_{G\,struc} = 200 \begin{bmatrix} 0.01606 & -0.00096 & 0 \\ -0.00096 & 0.00096 & -2.4 \\ 0 & -2.4 & 16{,}000 \end{bmatrix}$$

$$P_G = \text{Global Structural Nodal Force Vector} = \begin{Bmatrix} -10 \\ 0 \\ 0 \end{Bmatrix}$$

$$\underbrace{200 \begin{bmatrix} 0.01606 & -0.0096 & 0 \\ -0.00096 & 0.00096 & -2.4 \\ 0 & -2.4 & 16{,}000 \end{bmatrix}}_{K_G} \underbrace{\begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \theta_3 \end{Bmatrix}}_{\Delta_G} = \underbrace{\begin{Bmatrix} -10 \\ 0 \\ 0 \end{Bmatrix}}_{P_G}$$

$$\Rightarrow \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{Bmatrix} = \frac{1}{200} \begin{Bmatrix} -688.516 \\ -1101.62 \\ -0.00083 \end{Bmatrix} = \begin{Bmatrix} -3.4426 \\ -5.5081 \\ -0.00083 \end{Bmatrix} \begin{array}{l} mm \\ mm \\ rad \end{array}$$

Answer ✓

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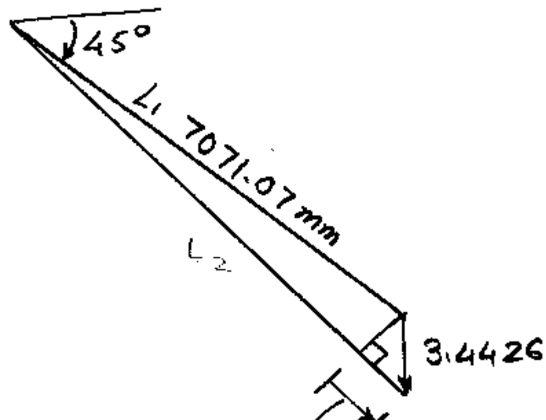
QNo 3

Tension in Truss Element

Calclulate change in length



$\Delta L = 3.4426 \times \cos 45°$

$= 2.4343$ mm.

$$\text{Axial Force} = \frac{AE}{L} \cdot \Delta L$$

$$= \frac{200 \times 200}{7071.07} \times 2.4343$$

$$= 13.77 \text{ KN Tension.}$$

Alternately

$$\{\Delta'\}_{Local} = T \; \Delta_{Global}$$

	x	Y
x'	0.7071	−0.7071

$\underbrace{\qquad\qquad}_{\gamma}$

$$= \lfloor 0.7071 \quad -0.7071 \rfloor \begin{Bmatrix} 0 \\ -3.4426 \end{Bmatrix}$$

$$\Delta = 2.4343 \text{ mm}$$

$$P = \frac{AE}{L} \cdot \Delta = \frac{200 \times 200}{7071.07} \times 2.4343 = 13.77 \text{ KN (Tension)}$$

Answer OK

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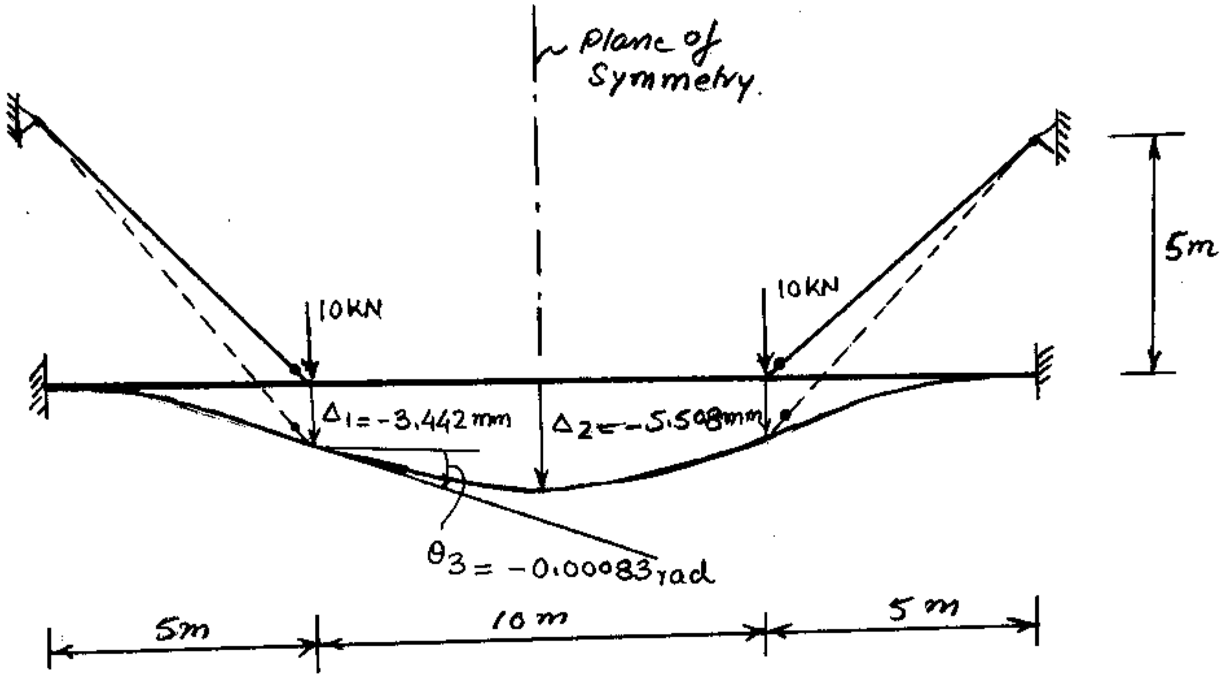
Q No. 3



Deformed Shape