Approximate Analysis

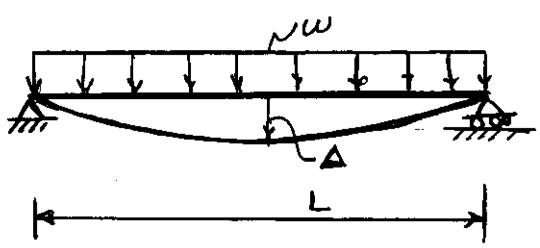
Approximate Analysis is a technique for obtaining forces and moments in a structure using simple calculations such that the forces and moments are reasonably accurate. The method is based on suitable assumptions regarding the the way forces are distributed in a structure and the way it deforms.

- Its primary use is to check the detailed engineering analysis and design carried out.
- It can also be used to quikkly check adequacy of a structure or structural member.
- It is not meant to substitute detailed analysis and design

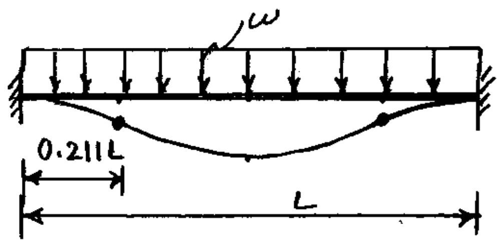
Approximate Analysis of Beams & Frames

Indeterminate beams and frames are analyzed by:

- making assumptions about location of pts of "contraflexure" (pts of zero bending moments)
- making assumptions about distribution of forces among members.



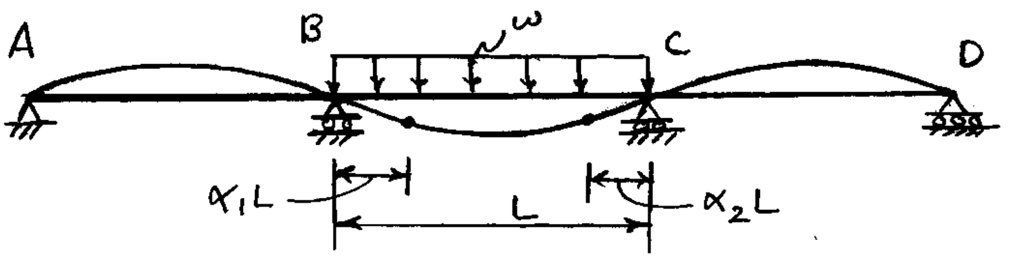
Simply supported Beam
(No Inflection Points)



Fixed End Beam
2 Inflection Pts.

Approximate Analysis

Approximate Analysis of Beams & Frames



Continuous Beam

In the continuous beam above the center span would have 2 inflection pts. The location of the inflection pts from pt B and C would depend upon the boundary conditions at pts B & C.

we know that:

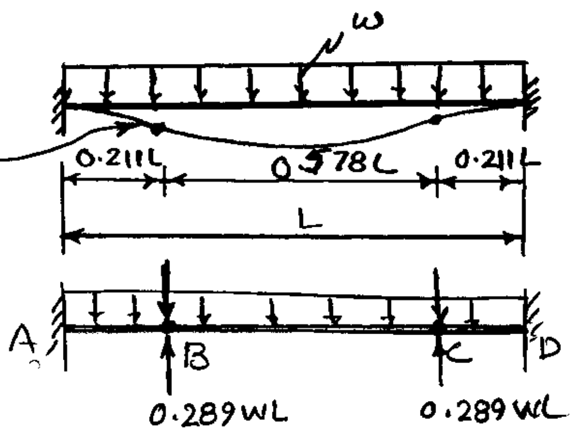
Location of inflection Pt for simply supported beam = 0.0L

" " " " for Fixed end beam = 0.211L

⇒ Estimated Location of inflection pt in the continuous beam above would be that it be somewhere between 0 → 0.211L. A reasonable assumption would be that

$$\alpha_1 L = \alpha_2 L = \left(\frac{0 + 0.211}{2}\right) L \quad 0.105 \simeq 0.1\,L$$

How Inflection Pts make Structure Determinate



Reaction @ pt B $= \dfrac{0.578wl}{2}$

$= 0.289\,wL$

Moment @ A $= -0.289wl \times 0.211\,l$

$+ \dfrac{w(0.211L)^2}{2}$

$= -(0.06098 + 0.0222)\,wl^2$

$= -0.0832\,wl^2 \simeq -\dfrac{wl^2}{12}$

Moment @ ℄ $= 0.289wl \times \dfrac{0.578L}{2} - \dfrac{w\left(0.578/_2\, L\right)^2}{2} = +0.0417wl^2 \simeq \dfrac{wl^2}{24}$

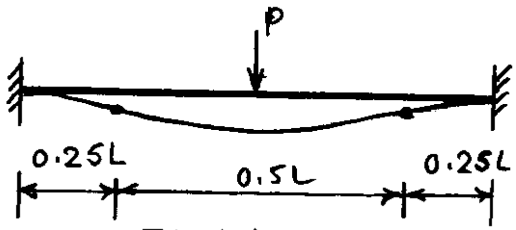
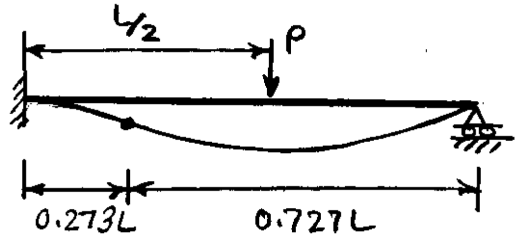
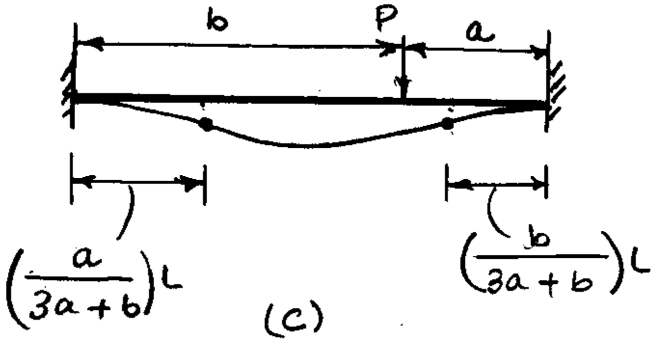


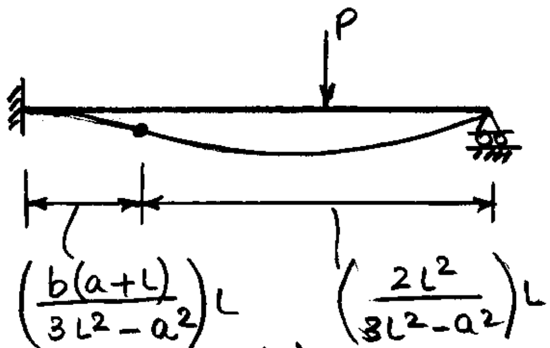
Fig. (a)



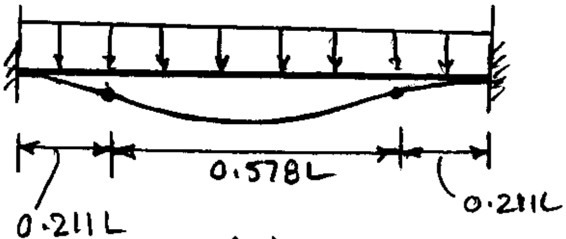
(b)



(c)

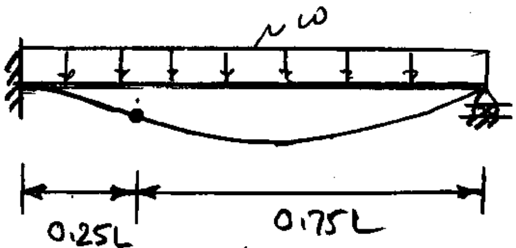


(d)



(e)

Fixed End Beam



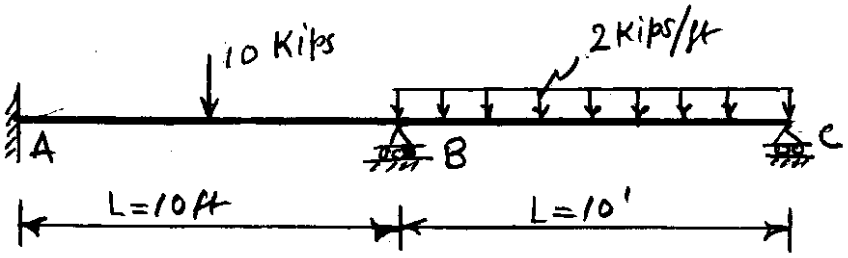
(f)

Proped Cantilever Beam

Location of Inflection Pts in Beams

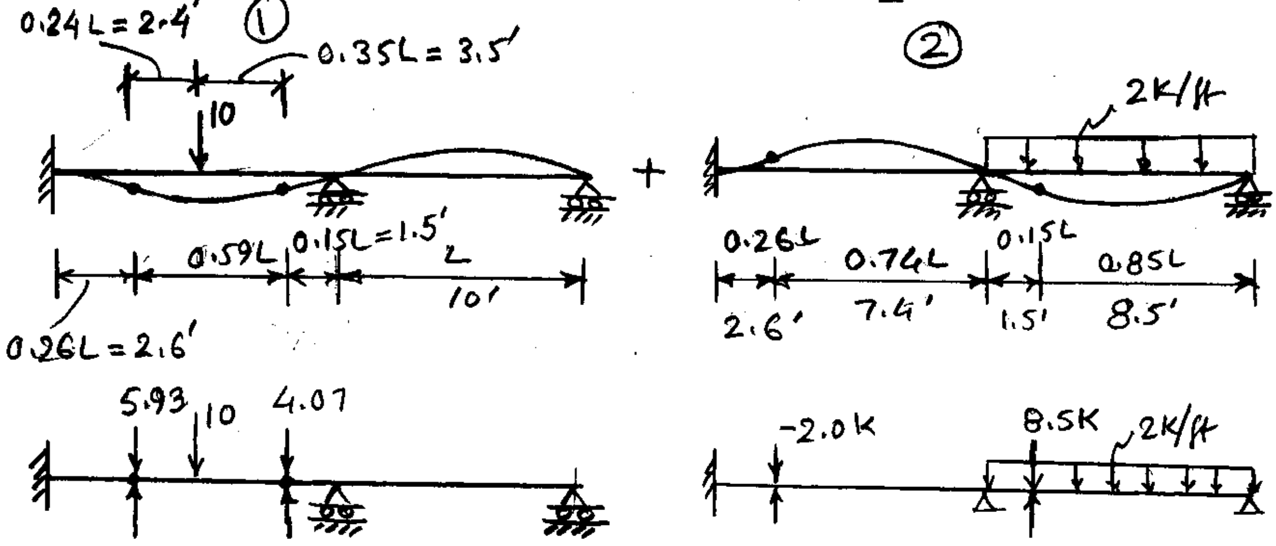
Example Problem

Analyze the beam shown below by Approximate Analysis.

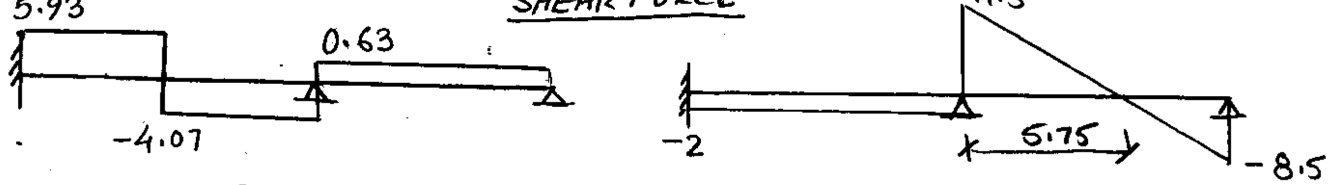


=

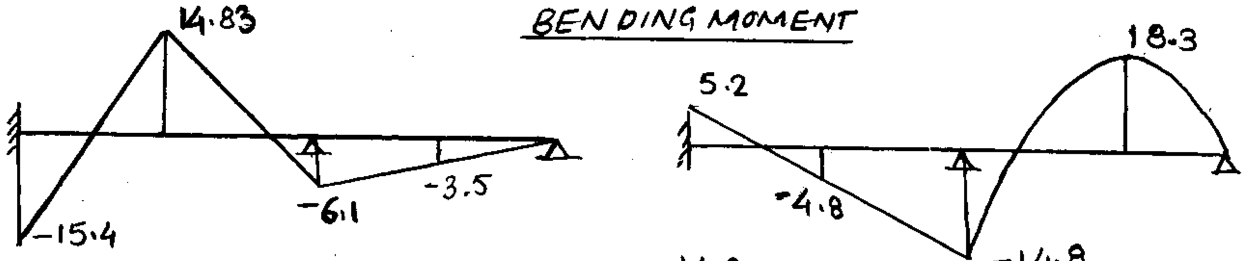
Splitting the problem into 2 cases we have

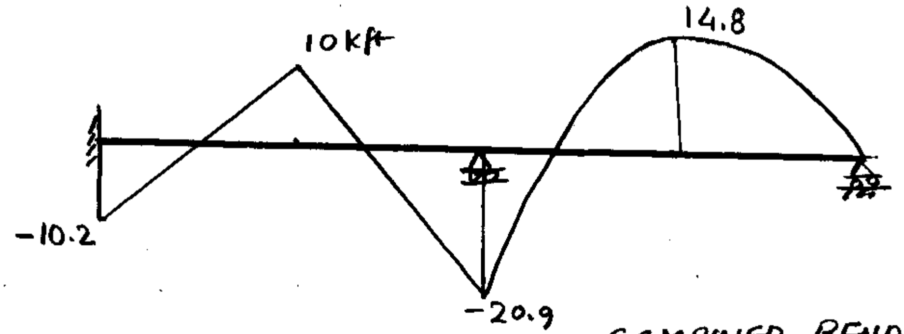


SHEAR FORCE



BENDING MOMENT





COMBINED BENDING MOMENT

Approximate Analysis

Example Problem

Inflection Pt Case ① $= \dfrac{0.25L + 0.273L}{2} = 0.262L$
(Ref Case a & b)
of Page 3
$\simeq 0.26L$

Inflection Pt Case 2 $= \dfrac{0.25L + 0L}{2} = 0.125L$
$\simeq 0.15L$
(Ref Case f & e
case of simply supported
beam)

Inflection pt near A $= \dfrac{(0.25 + 0.27)L}{2} = 0.26L$

Reactions @ inflection Pts
in Case ① respectively are $10 \times \dfrac{3.5}{(3.5+2.4)} = 5.93\,K \updownarrow$

and $10 \times \dfrac{2.4}{(3.5+2.4)} = 4.07\,K \updownarrow$

R_B Case ① $= \dfrac{4.07(10+1.5)}{10} = 4.71K \uparrow$

M_A Case ① $= -5.93 \times 2.6 = -15.4$ kip-ft

Moment under Pt Load Case ① $= 5.93 \times 2.4 = 14.23$ K-ft

Moment @ Pt B Case ① $= -4.07 \times 1.5 = -6.1$ K-ft

Moment @ Pt C $= 0$

Example Problem

Case 2

Shear @ 2nd Inflection pt $= \dfrac{2 \times 8.5'}{2} = 8.5$ kips

$R_C = 8.5$ kips ↑

$R_B = ?$

Taking moments at Inflection Pt 1

$$8.5 \times (7.4 + 1.5) + 2 \times 1.5 \times \left(7.4 + \frac{1.5}{2}\right) - R_B \times 7.4 = 0$$

$$\Rightarrow R_B = \frac{100.1}{7.4} = 18.5\,K \uparrow$$

$R_C =$

Reaction @ First Inflection pt $= R_A$

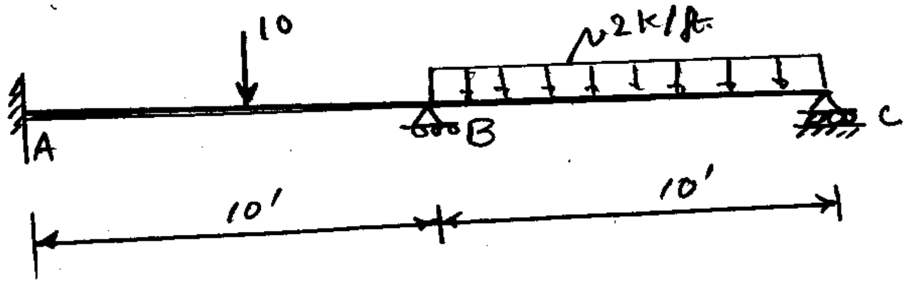
$$= 2 \times 10 - R_B - R_C = 20 - 13.5 - 8.5 = -2\,K \downarrow$$

$$M_A = 2 \times 2.6 = 5.2 \text{ kft}$$

$$M_B = -2.0 \times 7.4 = -14.8 \text{ kft}$$

Approximate Analysis

Example Problem



	A	BA	BC	CB
		0.571	0.429	1.0
		0.5	0.5	0.5
FEM	-12.5	$+12.5$	-16.67	$+16.67$
			-8.33	-16.67
	3.57	$+7.13$	$+5.36$	
	-8.93	$+19.63$	-19.64	0
R Load	5	5	10	10
R Momt	-1.07	$+1.07$	$+1.964$	-1.964
	$+3.93$	$+6.07$	$+11.964$	$+8.034$

$$FEM_{AB} = \frac{PL}{8} = \frac{10 \times 10}{8} = -12.5 \text{ K-ft}$$

$$FEM_{BC} = \frac{wl^2}{12} = \frac{2 \times 10^2}{12} = -16.67 \text{ K-ft}$$

$$K_{BA} = \frac{I}{L} = \frac{I}{10} = 0.1 I = K$$

$$K_{BC} \text{ Modified} = \frac{3}{4}\frac{I}{10} = 0.75 \times 0.1 I = 0.75K$$

$$K_{CB} = \frac{I}{10} = 0.1 I = K$$

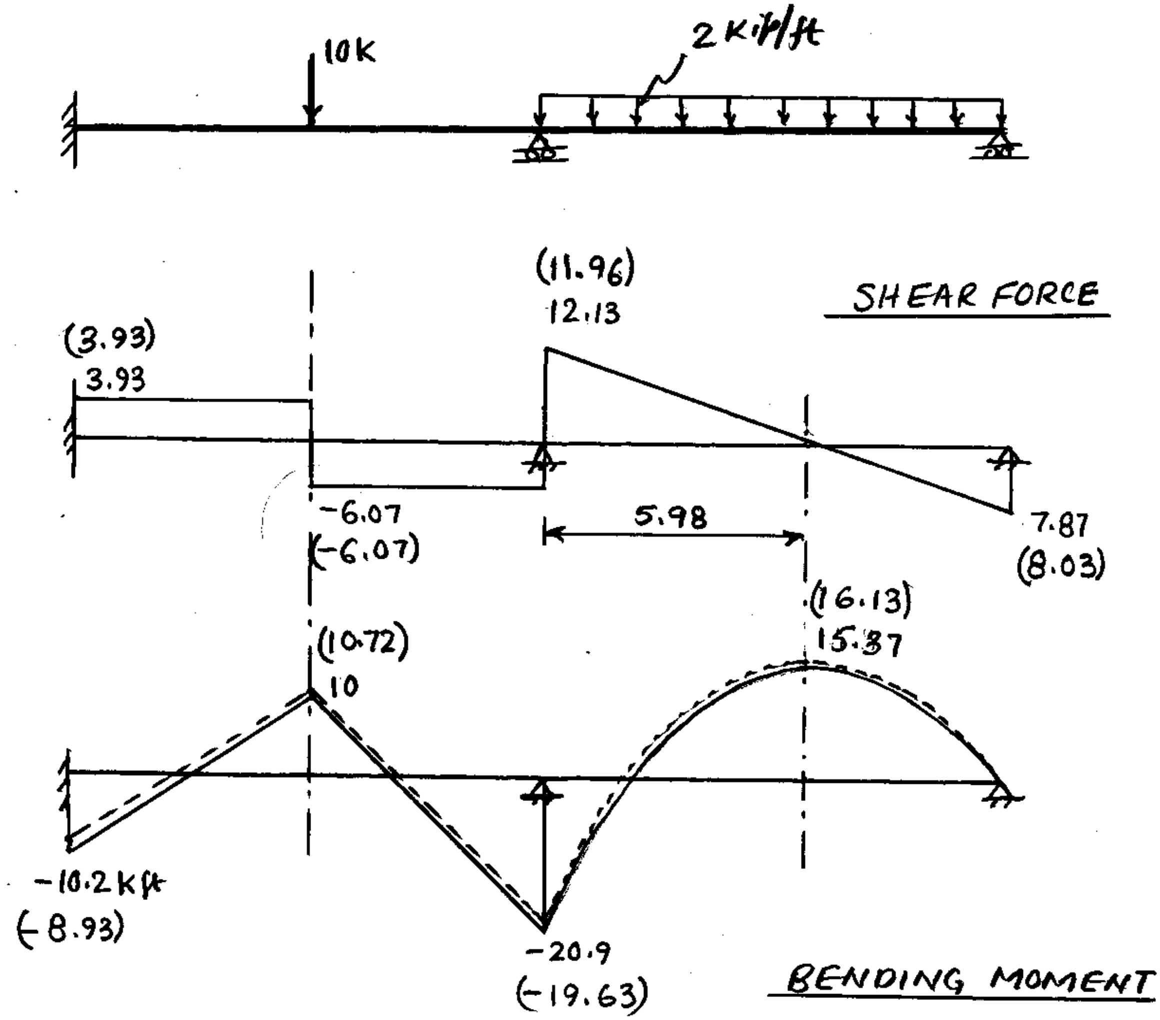
Distribution Factors

$$D_{BA} = \frac{K}{K + 0.75K} = 0.571$$

$$D_{BC} = \frac{0.75K}{K + 0.75K} = 0.429$$

$$D_{CB} = \quad = 1.0$$

Example Problem

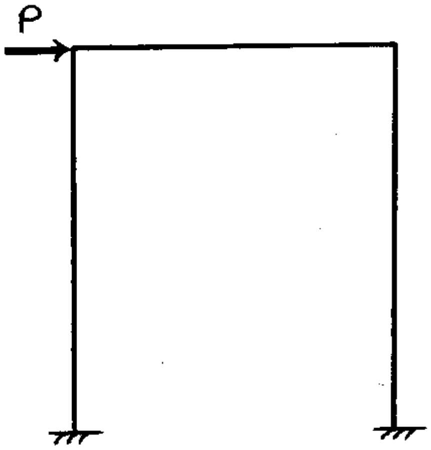


COMPARISON OF APPROXIMATE & EXACT SOLUTIONS

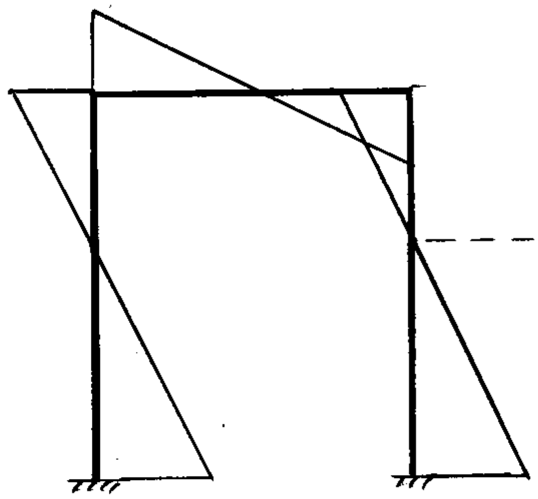
Approximate Analysis

Portal Frames

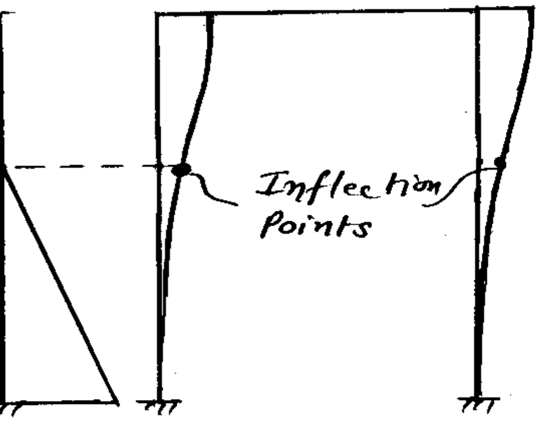
Portal Frames are quite efficient in transmitting and withstanding Lateral Forces. The Figure shown below shows the bending moment and the deflected shape of the portal frame.



Portal Frame



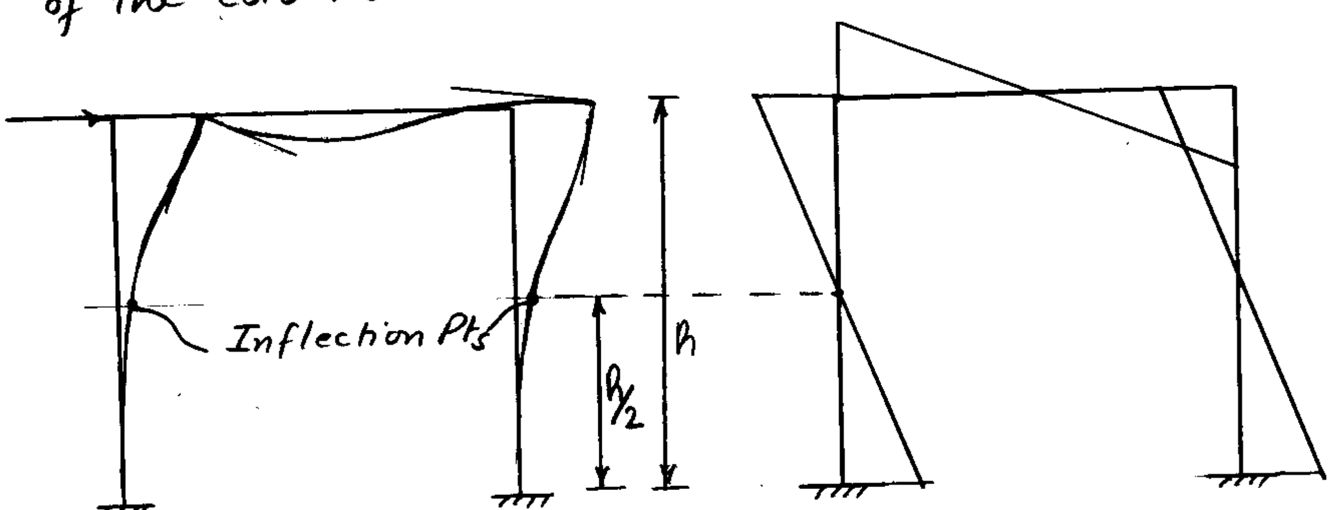
Bending Moment



Deflected Shape

Case of Very Stiff Girder

- If the girder is very stiff as compared to the columns the bending of the girder is negligible and the bending moment at the top of the columns is approximately the same as that at the base of the columns.
- The inflection pts in this case are at the mid-height of the columns

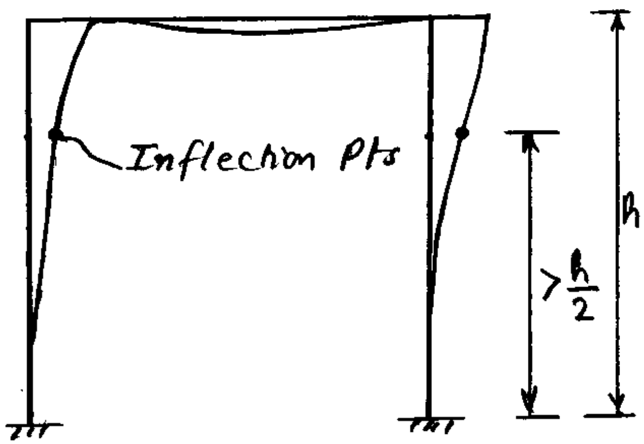


Case of Very Stiff Girder - Inflection Pts @ midheight

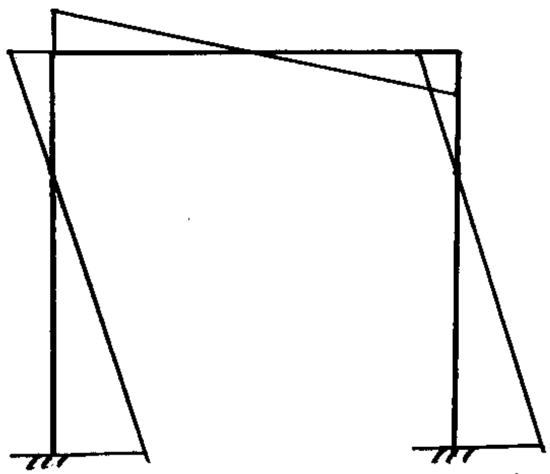
Portal Frames

Case of Flexible Girder

- In case of girder being quite flexible in comparison with the column, the pts of inflection lie near the top of the columns.



Deflected Shape



Bending Moment

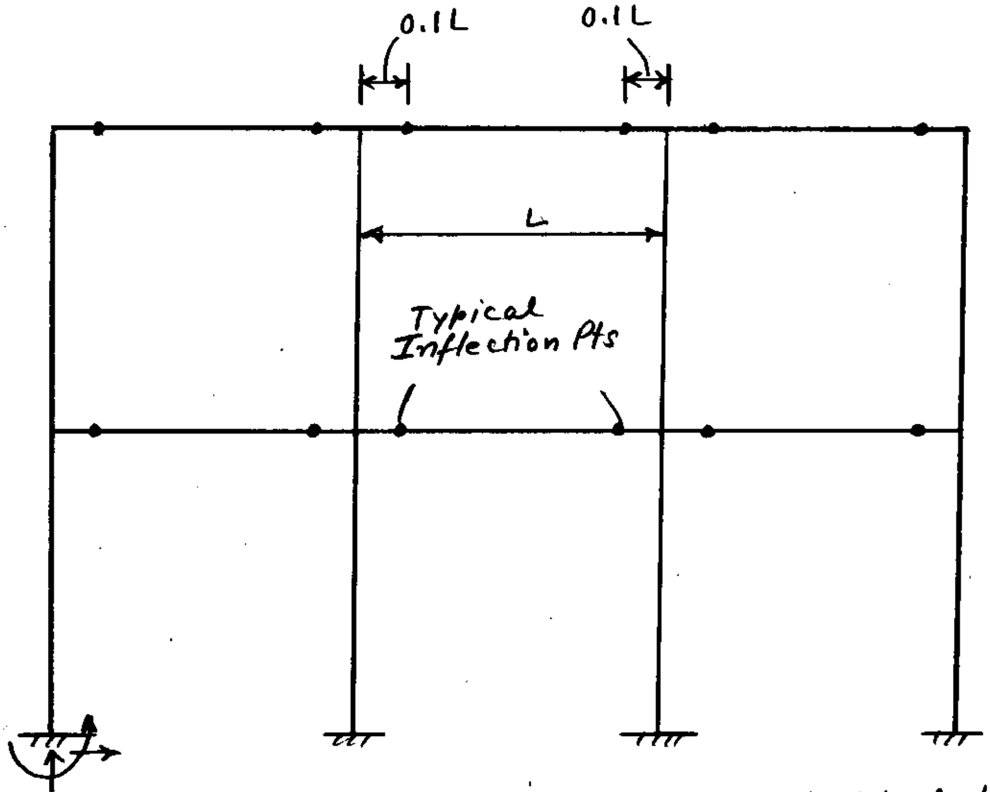
Case of very Stiff Girder – Inflection Pts near top of columns

Assumptions for Analysis of Portal Frames

1. Inflection Pts occur at the mid-height of the columns
2. The shear in each column is equal to half of the total applied shear to the portal.

Approximate Analysis

Building Frames Subjected to Vertical Forces only



Building Frame subjected to Vertical Loads only
Location of Inflection Pts $\simeq$ 0.1L from Columns

The frame is statically indeterminate:

number of members $= b = 14$

number of independent generalized reactions $= r = 3 \times 4 = 12$

number of joints $= n = 12$

Then,

Degree of Indeterminancy $= N = 3b + r - 3n$

$$= 3 \times 14 + 12 - 3 \times 12$$

$$= 54 - 36$$

$$= 18$$

Approximate Analysis

Building Frames Subjected to Vertical Forces Only

Assumptions

1. Inflection pts occur at a distance 0.1L from the columns
2. There are no axial forces developed in the girders.

- The first assumption yields 12 additional equations as there are 12 hinges/inflection pts.
- The second assumption yields 6 additional equations

Total Number of additional equations yielded due to simplifying assumptions = 18

$\Rightarrow$ Net indeterminancy = 18 − 18 = 0

$\Rightarrow$ Hence we see that the assumptions made make the building frame statically determinate

Approximate Analysis

Building Frames Subjected to Lateral Loads

The lateral loads such as wind load and earthquake loading can be transferred to the floor level joints in a frame model of a multi-storey building.

Following two methods may be used for approximate analysis of laterally loaded frames:

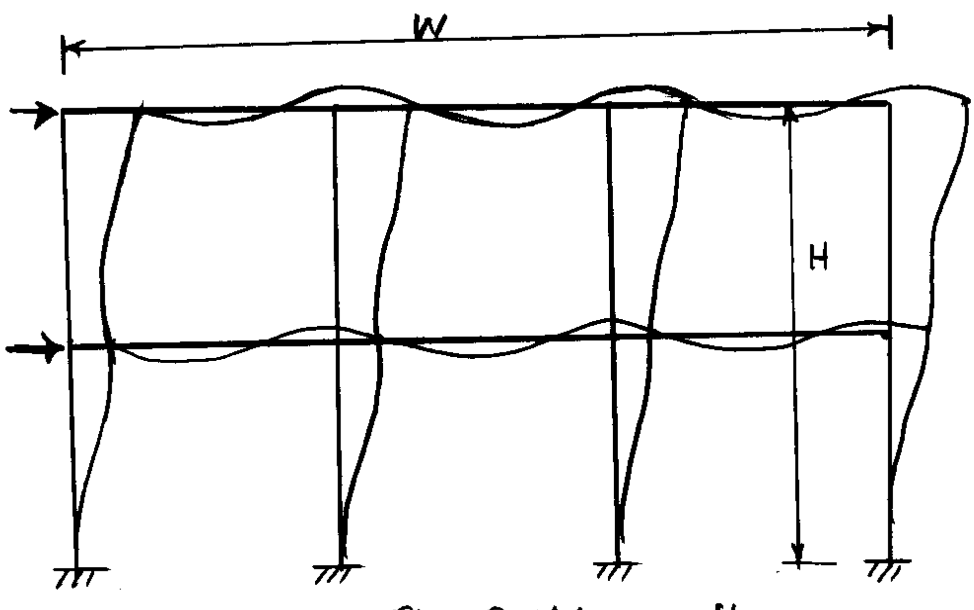
1. The Portal Method – Applicable for frames with $\frac{\text{Height}}{\text{Width}} \leq 1.0$

2. Cantilever Method – Applicable for frames with $\frac{\text{Height}}{\text{Width}} > 1.0$

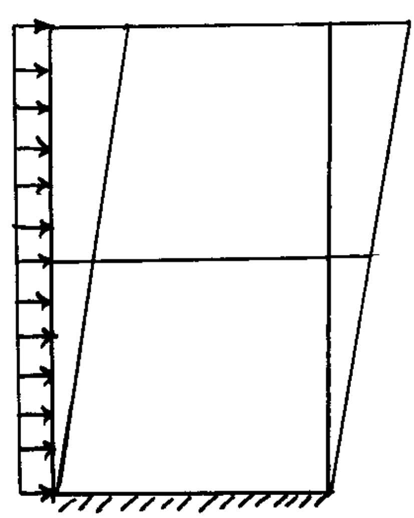
Behaviour of Low-rise Buildings

The buildings for which height to width ratio is less than 1.0 are most suitable for analysis by the "Portal Method".

Under lateral Loading the behaviour of such buildings is simillar to shearing behaviour of a solid block as shown in figure below:



Low-Rise Building – $\frac{H}{W} \leq 1.0$



Pure-Shear Response

Approximate Analysis

Low-Rise Buildings

Low-Rise buildings can be approximately analyzed by the "Portal Method"

Basic assumptions involved in the Portal Method are:

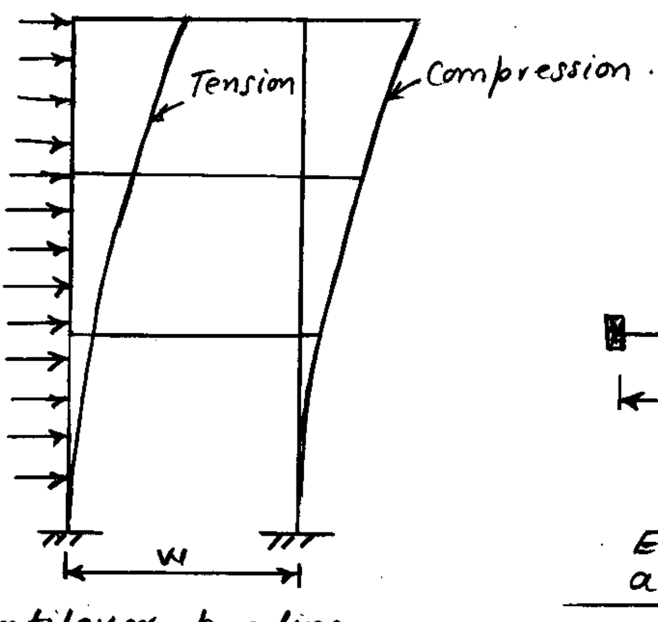
1) Lateral shear in each storey is equally distributed in each panel. This means that the interior columns will carry twice the shear as the exterior columns.

2.) There is a point of inflection at the center of each girder

3) There is a point of inflection at the center of each rigidly connected girder. This assumption does not apply if the column bases are pinned.

Behaviour of Medium Height and High-Rise Buildings

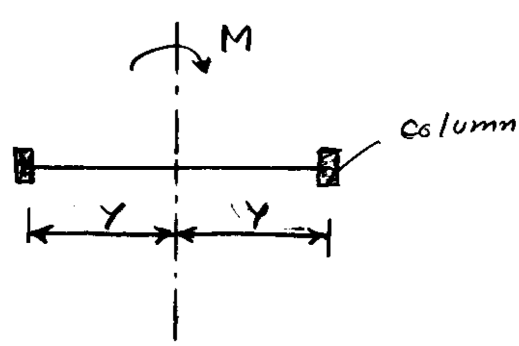
The medium-height and high-rise buildings is simillar to a cantilever in bending as shown below. The axial forces/stresses in columns can be estimated using elastic bending theory.



Cantilever bending of high-rise Bldgs

Stress in Column $= \sigma = \frac{M}{I} y$

Force in Col $= A\sigma$



Estimation of Column axial Forces

Approximate Analysis

Medium-Rise to High-Rise Bldgs.

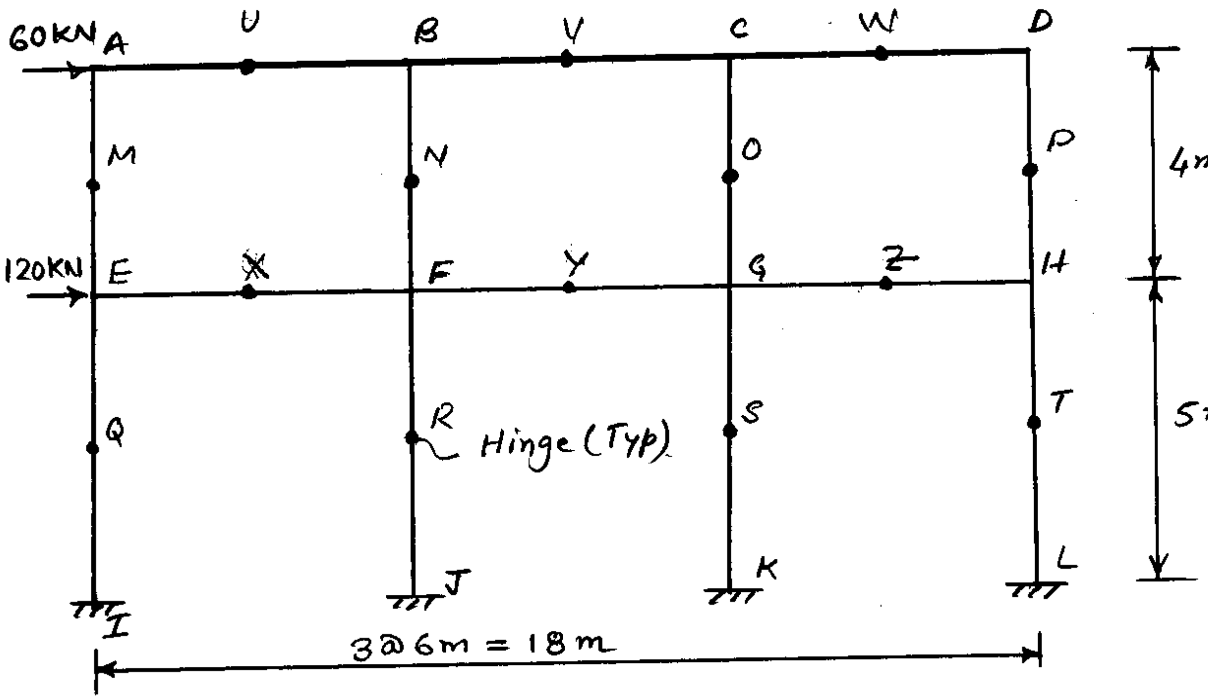
Medium-rise and high-rise buildings subjected to lateral loadings can best be approximately analyzed using the "Cantilever Method"

Basic assumptions of "Cantilever Method" are:

1. The building behaves like a free standing cantilever. The axial forces in columns can be determined using elastic bending theory
2. The assumption regarding location of inflection pts in girders and columns are the same as in the case of the "Portal Method"
3. In cantilever method the column axial forces are determined first and then other moments and forces are estimated.

Example Problem - Portal Method

Analyze the laterally loaded frame shown below by approximate analysis.



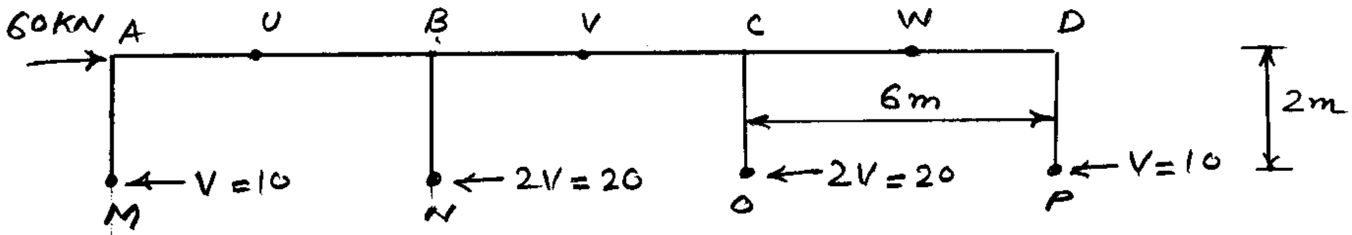
Height = H = 5+4 = 9 m

Width = W = 18 m

$$\frac{\text{Height}}{\text{Width}} = \frac{H}{W} = \frac{9}{18} = \frac{1}{2} < 1.0 \Rightarrow$$ Low-rise Bldg, Portal Method applicable

Analysis - Storey ②

Isolating 1st storey at inflection pts MNOP



Shear @ Exterior Column = ?

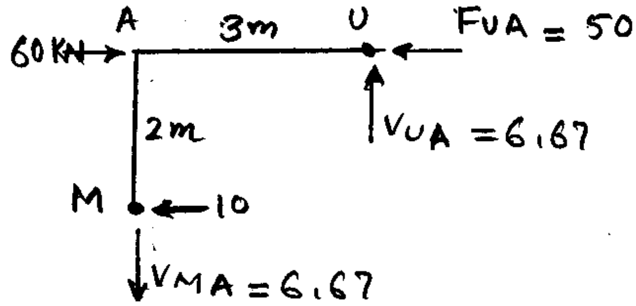
$\Sigma H = 0 \Rightarrow V + 2V + 2V + V = 60$

$\Rightarrow 6V = 60$

$\Rightarrow V = 10\text{ KN}$

Example – Portal Method

2nd Storey – Frame MAU



$\Sigma H = 0$

$\Rightarrow F_{UA} = 50\,KN$

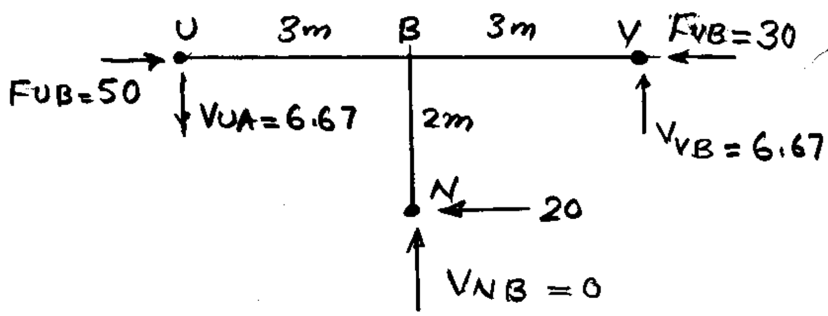
Takeng moments @ A

$10 \times 2 = 3\,V_{UA}$

$\Rightarrow V_{UA} = \frac{20}{3} = 6.67\,KN \uparrow$

$\Rightarrow V_{MA} = \quad = 6.67\,KN \downarrow$

Frame UBVN



$\Sigma H = 0$

$50 - 20 = F_{VB}$

$F_{VB} = 30\,KN$

Moment @ B

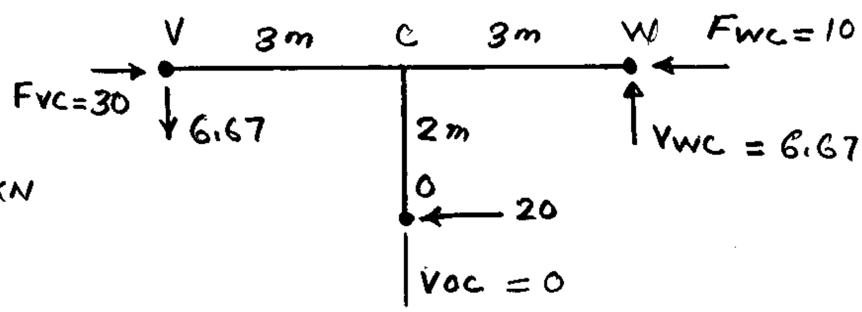
$-6.67 \times 3 + 20 \times 2 - 3 \times V_{VB} = 0$

$\Rightarrow V_{VB} = \frac{20}{3} = 6.67 \uparrow$

$\Sigma V = 0$

$\Rightarrow V_{NB} = \quad = 0$

Frame VCWO



$\Sigma H = 0$

$\Rightarrow F_{WC} = 30 - 20 = 10\,KN$

Moment @ C

$-6.67 \times 3 + 20 \times 2 - 3\,V_{WC} = 0$

$\Rightarrow V_{WC} = \frac{20}{3} = 6.67 \uparrow$

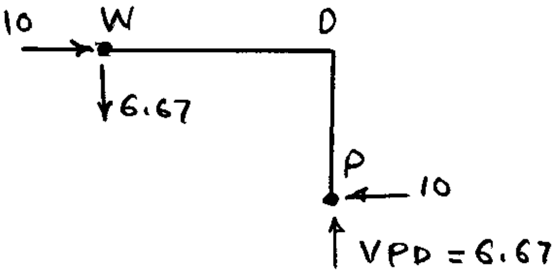
$\Sigma V = 0$

$\Rightarrow V_{OC} = \quad = 0$

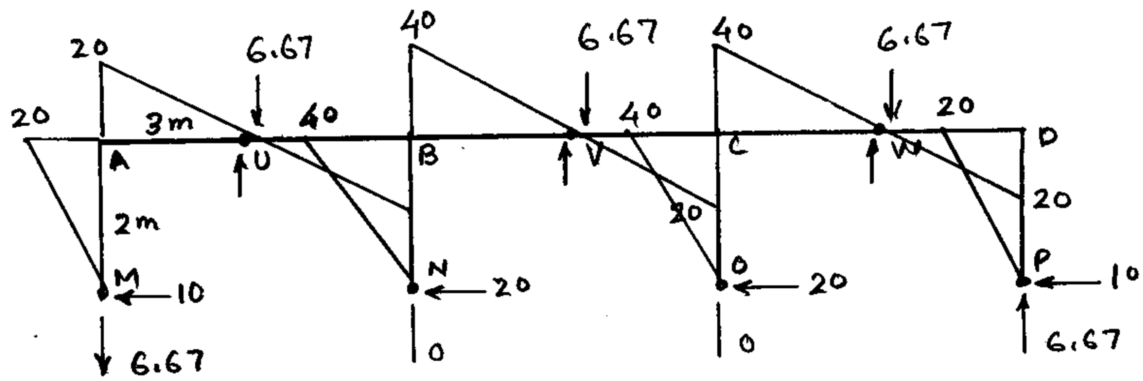
2nd Storey

Frame WDP

$V_{PD} = 6.67 \uparrow$



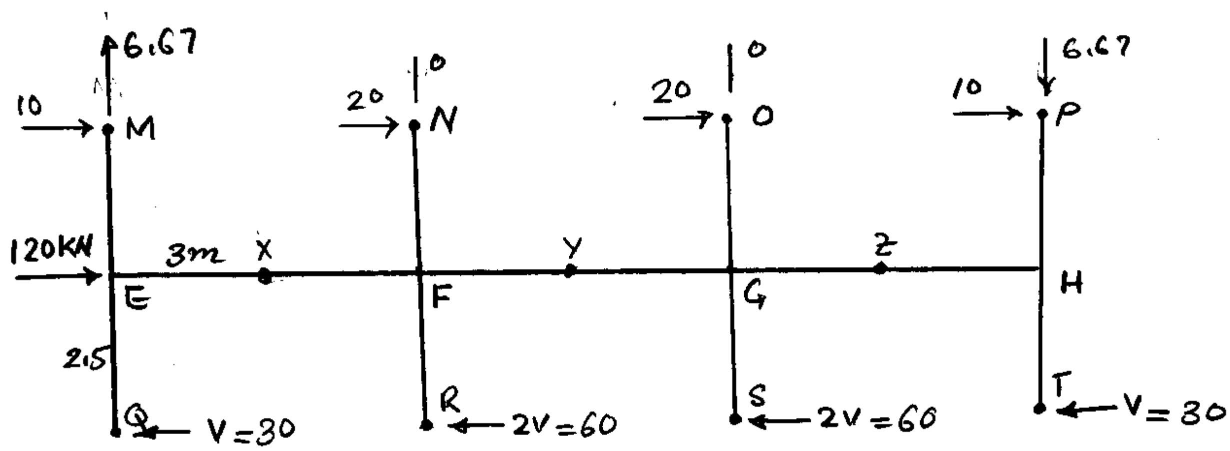
Bending Moment 2nd Storey



Bending Moments 2nd Storey

1st Storey

Isolate 1st Storey as shown below:



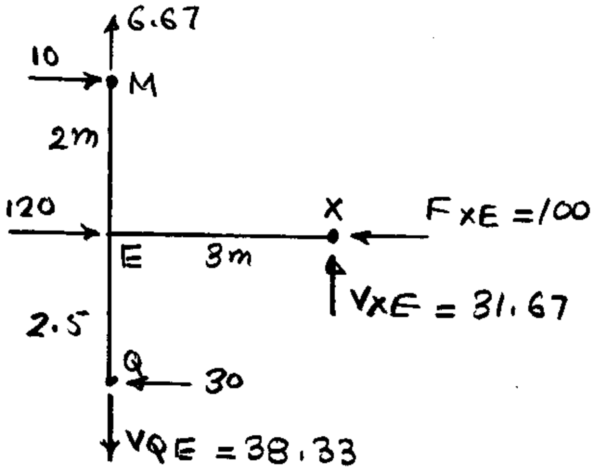
$\Sigma H = 0$

$6V = 60 + 120$

$V = 30$

1st Storey

Frame MEXQ



$\Sigma H = 0$

$F_{XE} = 120 + 10 - 30 = 100\ KN$

Moment @ E

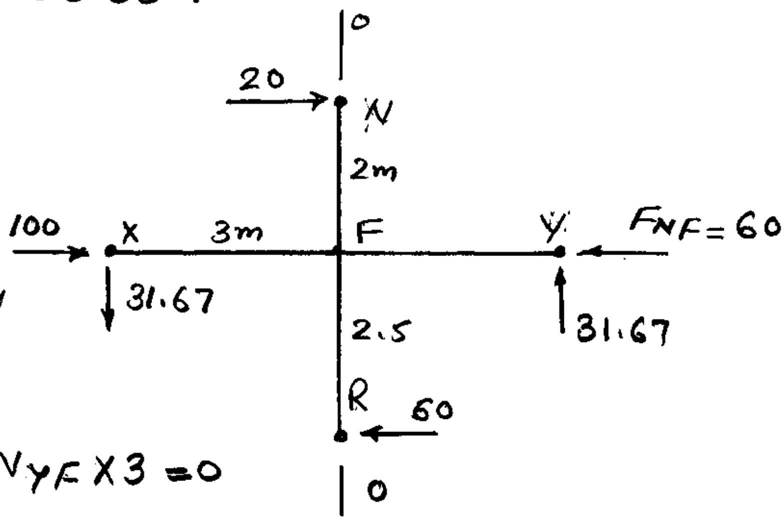
$10 \times 2 + 30 \times 2.5 = 3\, V_{XE}$

$\Rightarrow V_{XE} = \dfrac{95}{3} = 31.67 \uparrow$

$\Sigma V = 0$

$\Rightarrow V_{QE} = 31.67 + 6.67 = 38.33 \downarrow$

Frame XNYR



$\Sigma H = 0$

$F_{NF} = 100 + 20 - 60 = 60\ KN$

Moments @ F

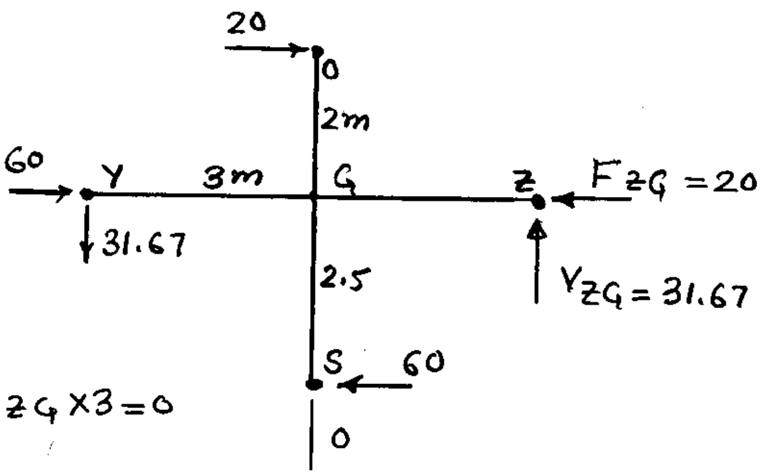
$-31.67 \times 3 + 60 \times 2.5 + 20 \times 2 - V_{YF} \times 3 = 0$

$\Rightarrow V_{YF} = \dfrac{95}{3} = 31.67$

$\Sigma V = 0$

$\Rightarrow V_{RF} = 0$

Frame YOZS



$\Sigma H = 0$

$F_{ZG} = 60 + 20 - 60 = 20$

Moment @ G.

$-31.67 \times 3 + 60 \times 2.5 + 20 \times 2 - V_{ZG} \times 3 = 0$

$V_{ZG} = \dfrac{95.0}{3} = 31.67\ KN$

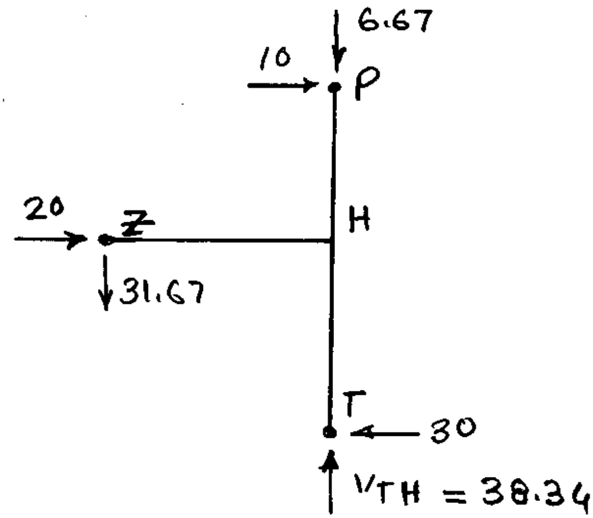
$\Sigma V = 0$

$\Rightarrow V_{SG} = 0$

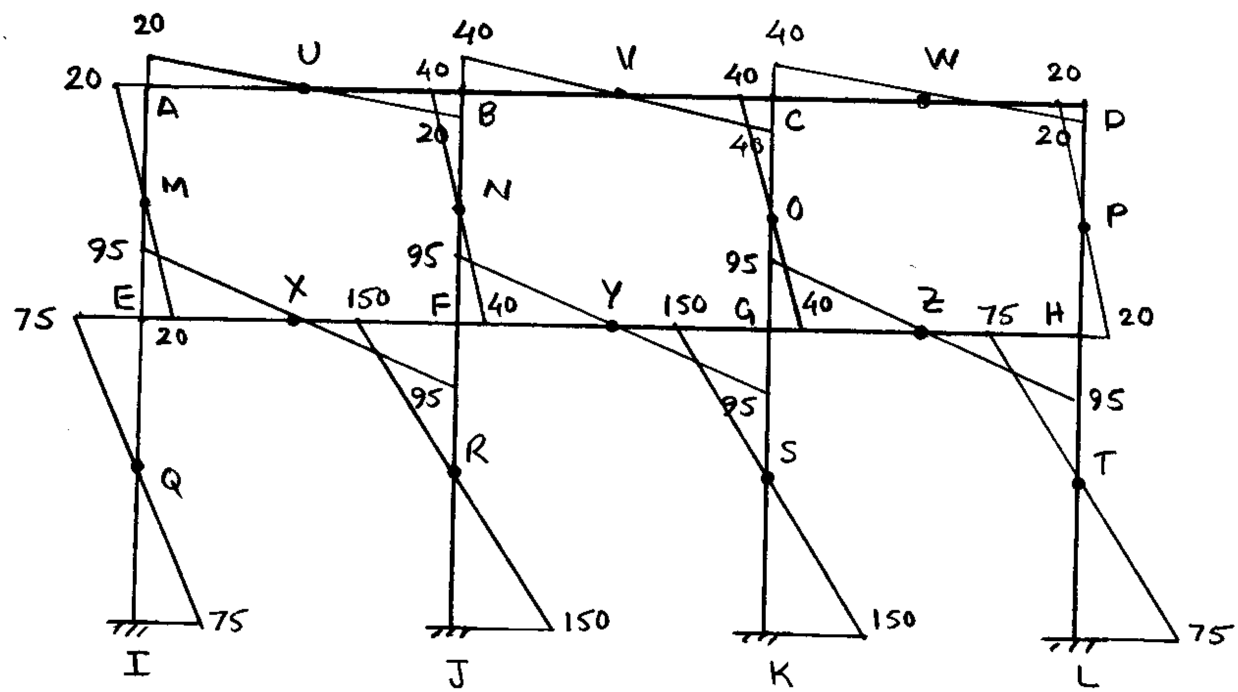
1st-Storey

Frame ZHPT

$V_{TH} = 31.67 + 6.67 = 38.34$



FRAME BENDING MOMENT



APPROXIMATE BENDING MOMENT

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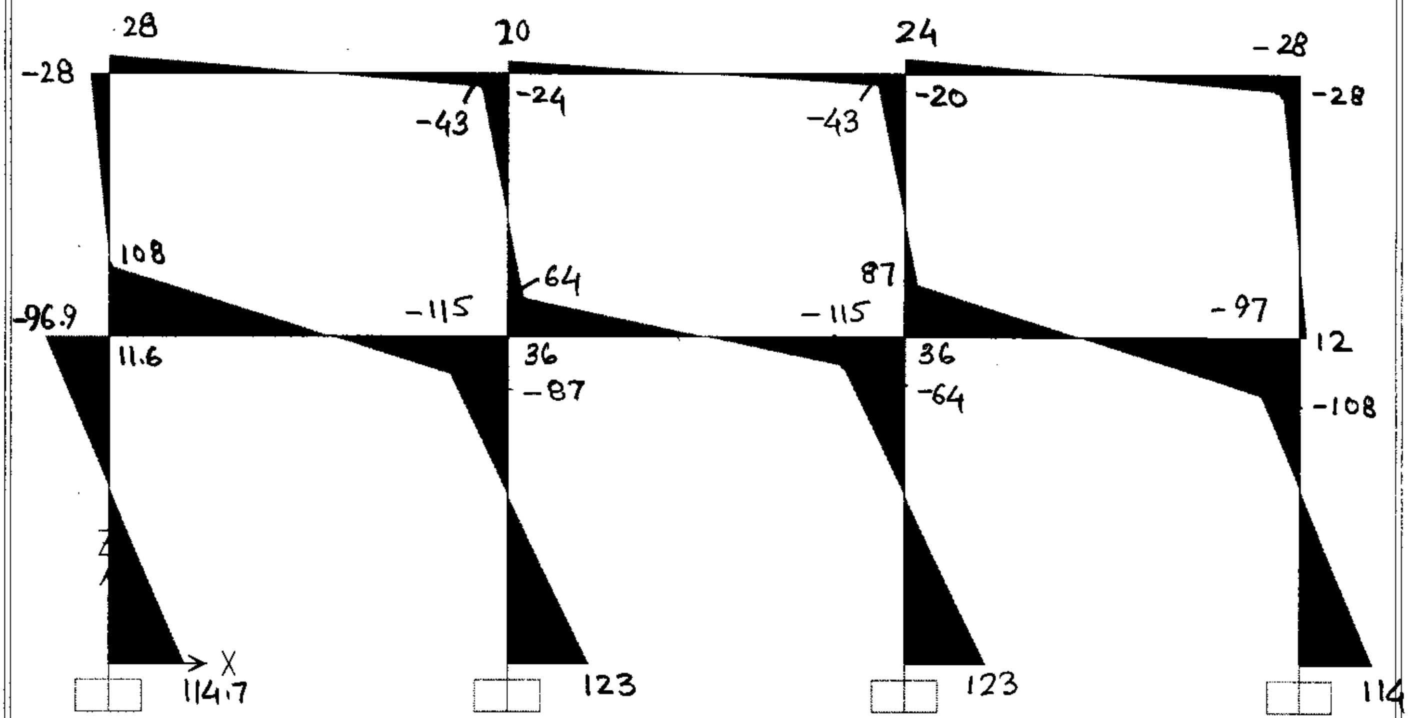
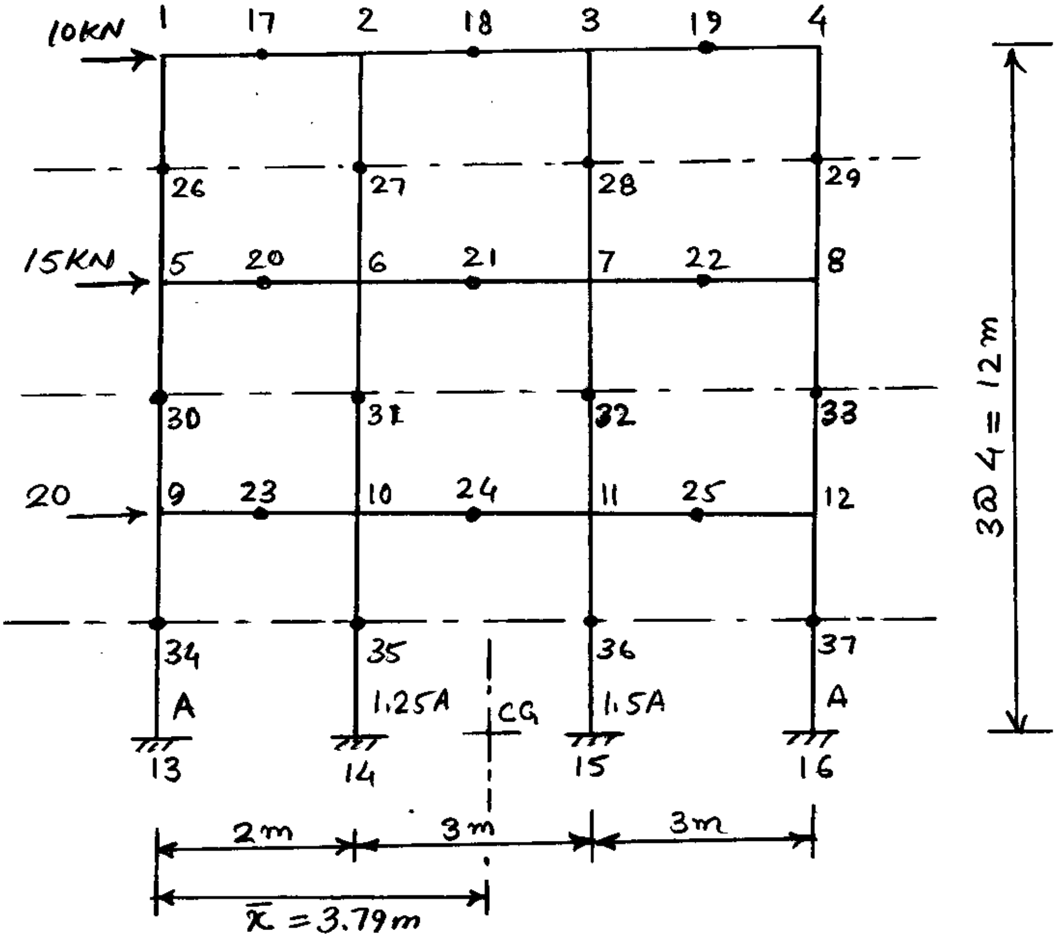


BENDING MOMENTS FROM SAP2000

Moment of Inertia of Girders is 3 Times that of Columns.

Example Problem - Cantilever Method

Analyise the building frame shown below using the Cantilever Method of Approximate Analysis. Areas of Columns are A, 1.25A, 1.5A and A respectively.



Soln.

Building Height = H = 12 m

" Width = W = 8 m

$$\frac{\text{Height}}{\text{Width}} = \frac{H}{W} = \frac{12}{8} = 1.5 > 1.0 \Rightarrow$$ Bldg. may be Analyzed by Cantilever Method.

Find C.G of the frame:

Taking moments @ col. line 1-5-9-13

$$(A + 1.25A + 1.5A + A)\,\bar{x} = 1.25A \times 2 + 1.5A \times 5 + A \times 8$$

$$\Rightarrow \bar{x} = \frac{1.25A \times 2 + 1.5A \times 5 + A \times 8}{(A + 1.25A + 1.5A + A)} = \frac{18A}{4.75A} = \underline{3.79m}$$

Example Problem – Cantilever Method

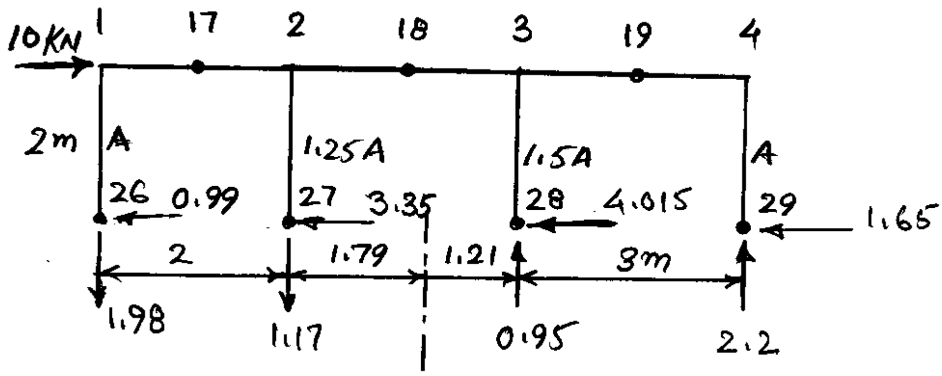
Moment of Inertia of Column System $= \sum A d^2$

$$I_c = A\times(3.79)^2 + 1.25A\times(1.79)^2 + 1.5A\times(1.21)^2 + A\times(4.21)^2 = 38.289A$$

$$\sigma = \frac{M}{I_c} x$$

$$F_{col} = \frac{M}{I_c} . x . \text{area of Column.}$$

Isolating 3rd Storey



Determining axial Forces

$$M = 10\times 2 = 20 \text{ KN-m}$$

$$F_{1,26} = \frac{M}{I_c} . 3.79\times A = \frac{20}{38.289A} \times 3.79\times A = 1.98 \text{ KN} \downarrow$$

$$F_{2,27} = \frac{M}{I_c} 1.79\times 1.25A = \frac{20}{38.289A} \times 1.79\times 1.25A = 1.17 \text{KN} \downarrow$$

3.15↑

$$F_{3,28} = \quad = \frac{20}{38.289A} \times 1.21\times 1.5A = 0.95 \text{ KN}\uparrow$$

$$F_{4,29} = \quad = \frac{20}{38.289} \times 4.21\times A = 2.2 \text{ KN}\uparrow$$

3.15↓

Check

Take moments @ pt 17

$$V_{1,26}\times 2 = 1.98\times 1$$

$$\Rightarrow V_{1,26} = \frac{1.98}{2} = \quad = \overleftarrow{0.99 \text{ KN}}$$

Example Problem – Cantilever Method

Taking moments @ hinge 18

$$0.99 \times 2 - 1.98 \times 3.5 - 1.17 \times 1.5 + V_{2,27} \times 2 = 0$$

$$\Rightarrow \quad V_{2,27} = \frac{6.705}{2} = \quad = \overleftarrow{3.35 \text{ KN}}$$

Taking moments @ hinge 19

$$2.2 \times 1.5 - V_{4,29} \times 2 = 0$$

$$V_{4,29} = \frac{3.3}{2} = \quad = \overleftarrow{1.65 \text{ KN}}$$

Also,

$$(0.99 + 3.35 + V_{3,28}) \times 2 - 1.98 \times 6.5 - 1.17 \times 4.5 + 0.95 \times 1.5 = 0$$

$$2\,V_{3,28} = 8.03$$

$$V_{3,28} = \frac{8.03}{2}. \quad = \overleftarrow{4.015} \text{ KN.}$$

Check

$$V_{1,26} + V_{2,27} + V_{3,28} + V_{4,29} = 10$$

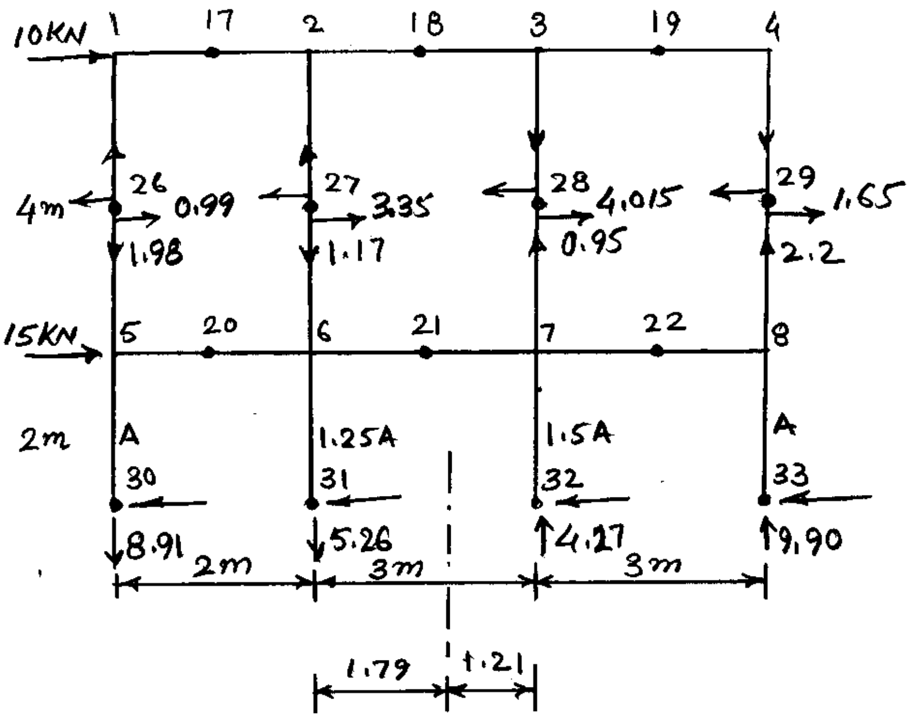
$$0.99 + 3.35 + 4.015 + 1.65 = 10$$

$$10.005 \simeq 10 \text{ KN} \quad \text{OK}$$

Approximate Analysis

Example Problem

Consider portion of the building above plane 30-31-32-33



Moment @ 30-31-32-33 $= 10 \times 6 + 15 \times 2 = 90$ KN-m

$F_{5,30} = \dfrac{90}{38.289A} \times 3.79A = \quad = 8.91 \downarrow$

$F_{6,31} = \dfrac{90}{38.289A} \times 1.79 \times 1.25A = \quad = 5.26 \downarrow$

$\left.\right\} 14.17 \downarrow$

$F_{7,32} = \dfrac{90}{38.289A} \times 1.21 \times 1.5A = \quad = 4.27 \uparrow$

$F_{8,33} = \dfrac{90}{38.289A} \times 4.21 \times A = \quad = 9.90 \uparrow$

$\left.\right\} 14.17 \uparrow$

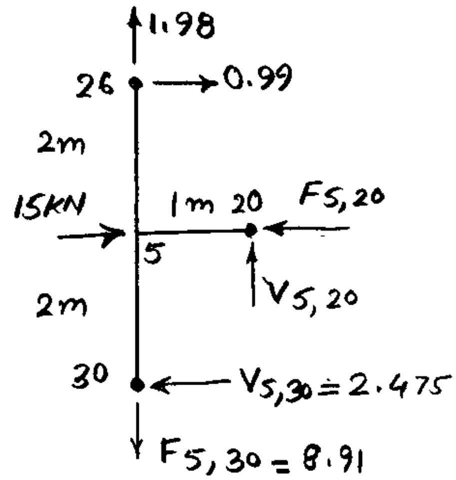
Check

Taking moments about pt 20

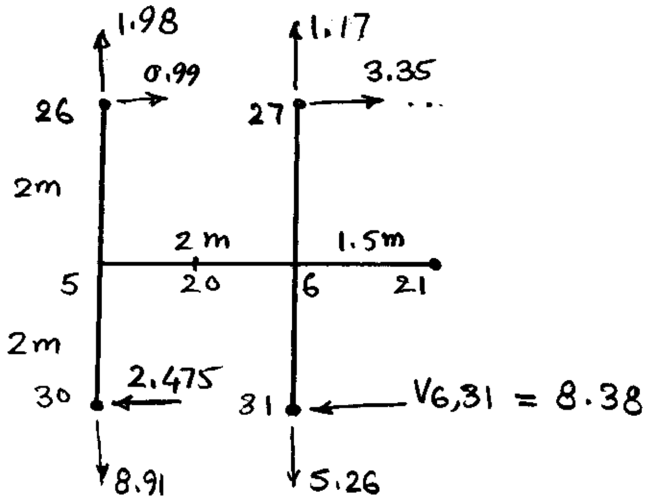
Frame 26-5-30-20

$-8.91 \times 1 + 1.98 \times 1 + 0.99 \times 2 + V_{5,30} \times 2 = 0$

$V_{5,30} = \dfrac{4.95}{2} = 2.475$ KN $\leftarrow$



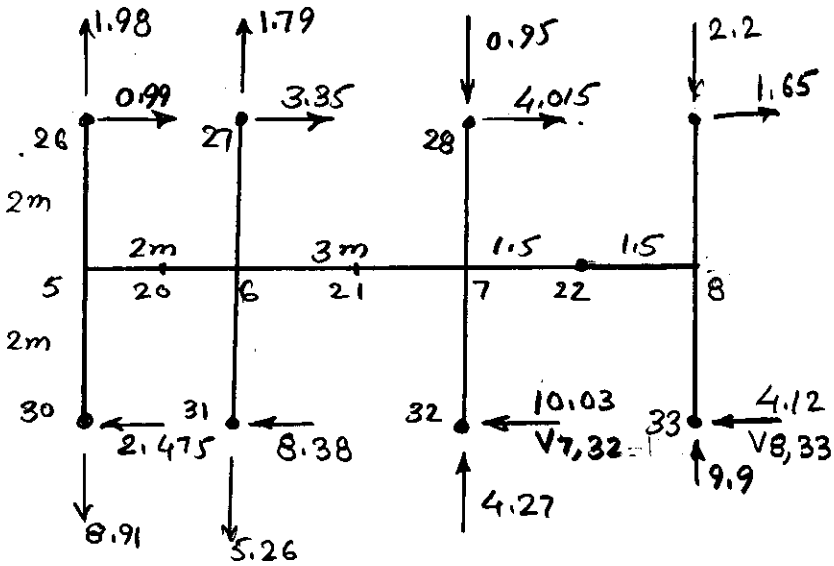
Example – Cantilever Method



Taking moments @ pt 21

$$(1.98-8.91)\times 3.5+(1.17-5.26)\times 1.5+(0.99+2.475+3.35)\times 2+2V_{6,31}=0$$

$$V_{6,31}=\frac{16.76}{2}= \qquad = \overleftarrow{8.38\text{ KN}}$$



Taking moments @ 22

$$(1.98-8.91)\times 6.5+(1.17-5.26)\times 4.5+(4.27-0.95)\times 1.5$$

$$+(2.475+8.38+0.99+3.35+4.015)\times 2+2\times V_{7,32}=0$$

$$\Rightarrow V_{7,32}=\frac{20.05}{2}=\overleftarrow{10.03\text{ KN}}$$

Taking moments @ 22 from Right side

$$(2.2-9.9)\times 1.5+1.65\times 2+2V_{8,33}=0$$

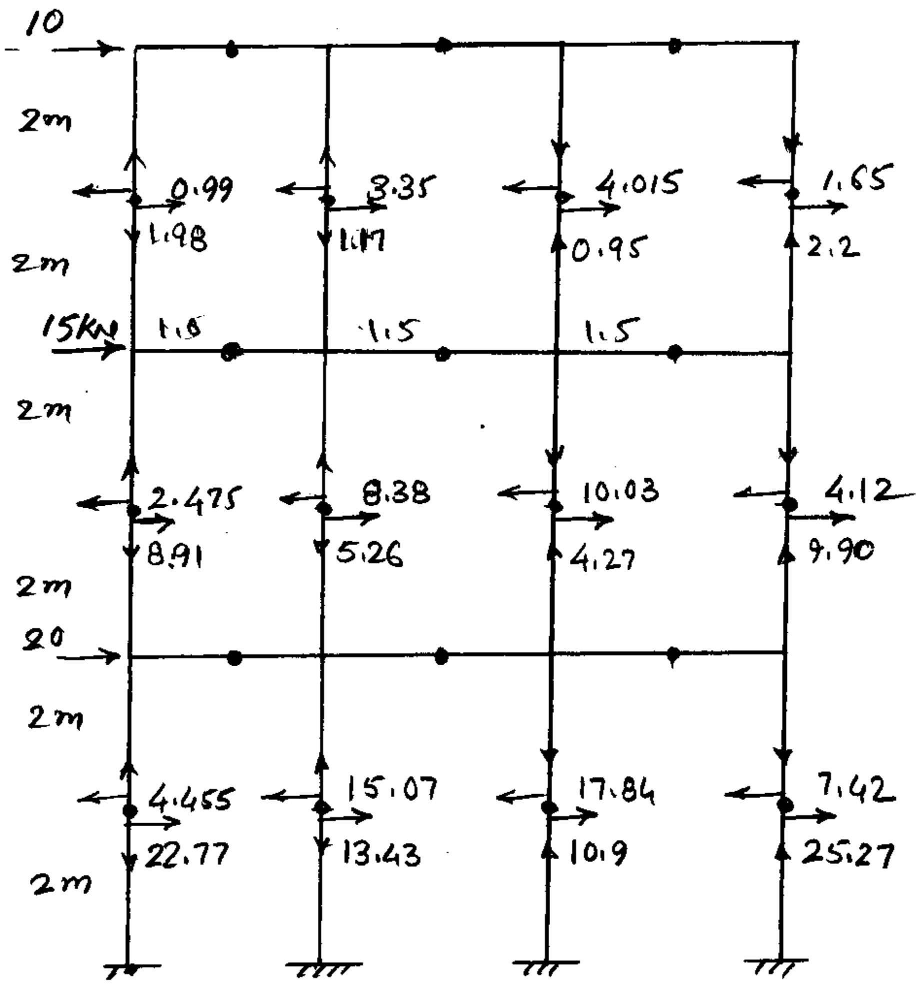
$$\Rightarrow V_{8,33}=\frac{8.25}{2}=\overleftarrow{4.12\text{ KN}}.$$

Check

$2.475+8.38+10.03+4.12=25$ KN OK Check

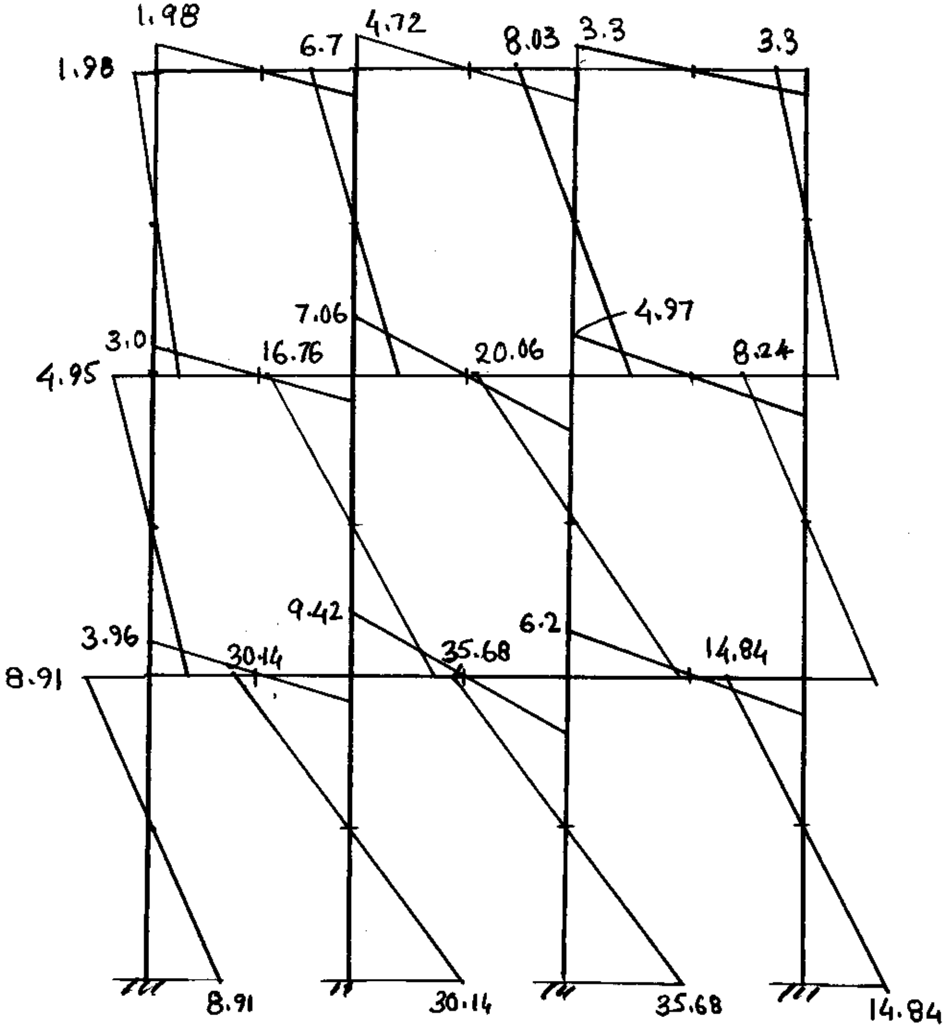
Example Problem – Cantilever Method

In simillar manner next we isolate Portion of the Frame between planes 30-31-32-33 and 34-35-36-37, and solve for unknown forces using equilibrium equations. For brevity these forces are not calculated and final forces are shown below:



FORCES AT HINGES/IN STRUCTURE

Example Problem - Cantilever Method



BENDING MOMENT DIAGRAM - APPROX. ANALYSIS

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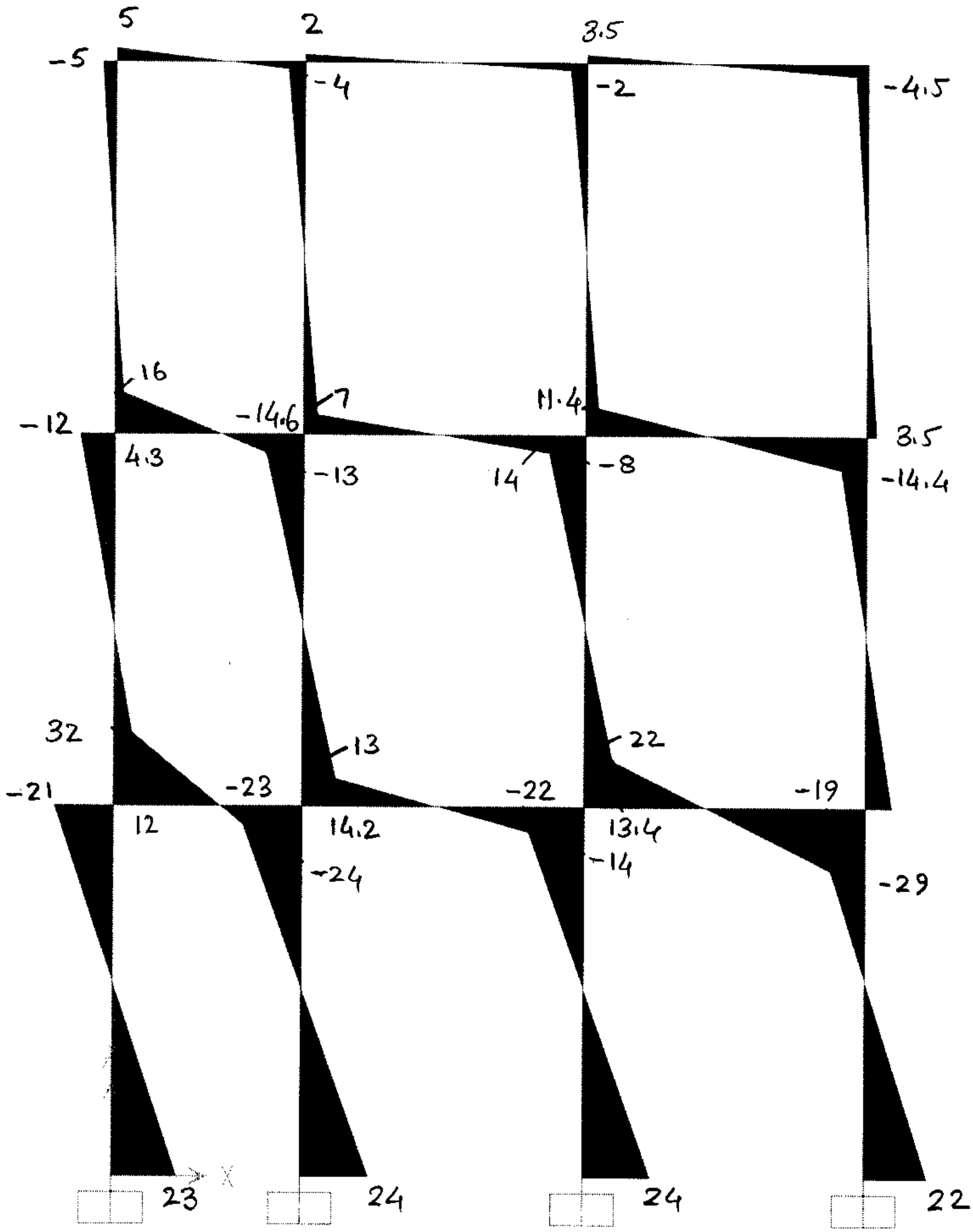

BENDING MOMENTS FROM SAP2000

Moment of Inertia of Girders is 3 times that of Columns.