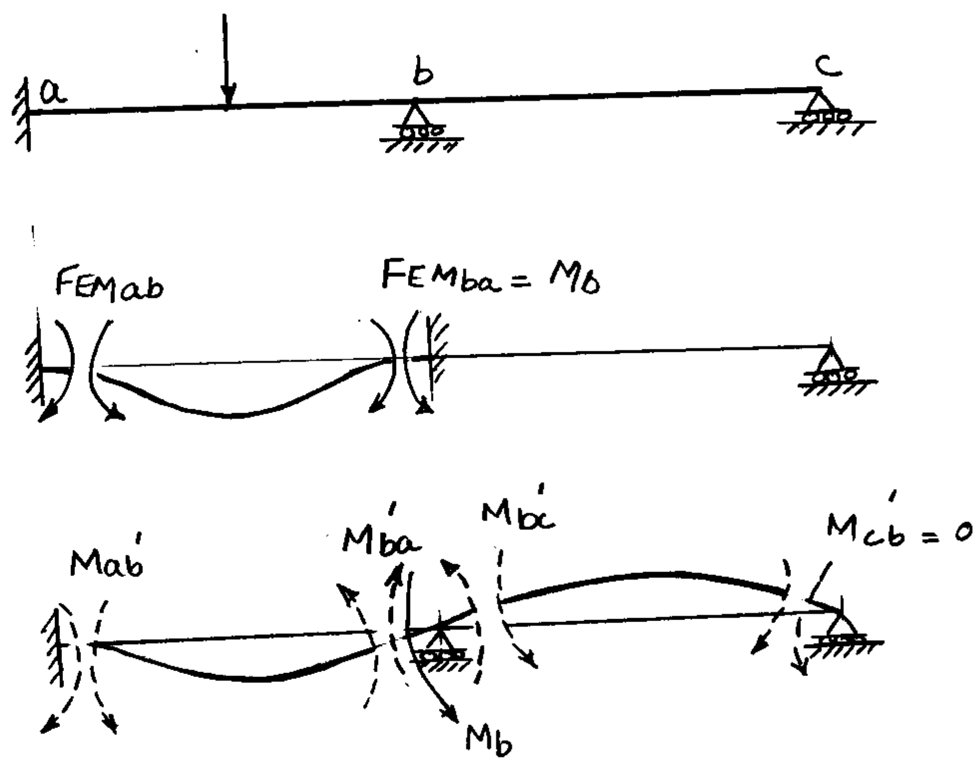
Modifications in Moment Distribution Method

When in a structure the exterior span is simply supported at the exterior support point, it is possible to introduce certain modifications that significantly cut down the amount of computations needed to analyze the structure.



Consider the structure shown above that has a exterior span having a roller at exterior end.
If joint "b" is initially clamped, Fixed end moments will $FEM_{ba} = M_b$ will develop there.
As the joint "b" is released, moments M'_{ba} and M'_{bc} will be induced alongwith carry over moment M'_{cb} at end "c".

However $M'_{cb} = 0$ as it is a roller end.

Modifications to Moment Distribution Method

From Slope-Deflection Equations we have:

$$M_{cb}' = 2EK_{bc}(2\theta_c + \theta_b) = 0 \qquad \text{——— (1)}$$

$$\Rightarrow \boxed{\theta_c = -\frac{\theta_b}{2}} \qquad \text{——— (2)}$$

Now

$$M_{bc}' = 2EK_{bc}(2\theta_b + \theta_c) \qquad \text{——— (3)}$$

From (2) and (3) we have

$$M_{bc}' = 2EK_{bc}\left(2\theta_b - \frac{\theta_b}{2}\right)$$

$$M_{bc}' = 3EK_{bc}\,\theta_b = 4EK_{bc}^{m}\,\theta_b \qquad \text{——— (4)}$$

where

K_{bc}^{m} = Modified Relative Stiffness of member bc $= \frac{3}{4}K_{bc}$

From Equilibrium considerations we have:

$$M_{ba}' + M_{bc}' = -M_b \qquad \text{——— (5)}$$

From Slope-Deflection Equations, Eqn (4) and (5) we have:

$$4E\theta_b\left(K_{ba} + K_{bc}^{m}\right) = -M_b \qquad \text{——— (6)}$$

Solving for "θ_b" we have:

$$\boxed{\theta_b = -\frac{M_b}{4E(K_{ba} + K_{bc})}} \qquad \text{——— (7)}$$

$$M_{ba} = 4EK_{ba}\,\theta_b \qquad \text{——— (8)}$$

Substituting value of "θ_b" from Eqn (7) into Eqn (8) and Eqn (4) we have

$$M'_{ba} = -\left(\frac{K_{ba}}{K_{ba} + K^m_{bc}}\right) M_b$$

and

$$M'_{bc} = -\left(\frac{K^m_{bc}}{K_{ba} + K^m_{bc}}\right) M_b \qquad \text{—— (9)}$$

Note that in above Distribution Factors, modified Relative Stiffness of member "bc" has been used.

General Form of Eqn (9) is

$$M'_{bi} = -M_b \frac{K'_{bi}}{\sum_j K_{bj}} = -M_b \cdot D'_{bi} \qquad \text{—— (10)}$$

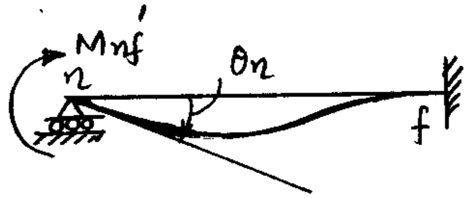
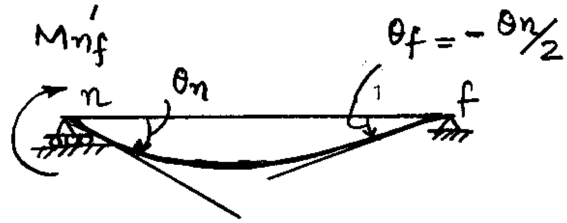
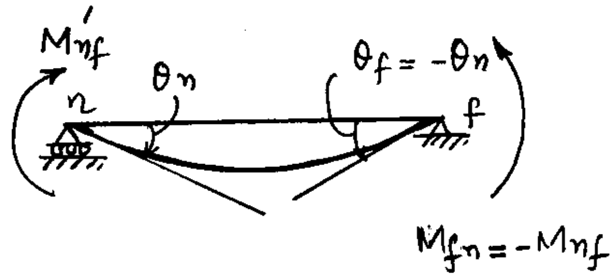
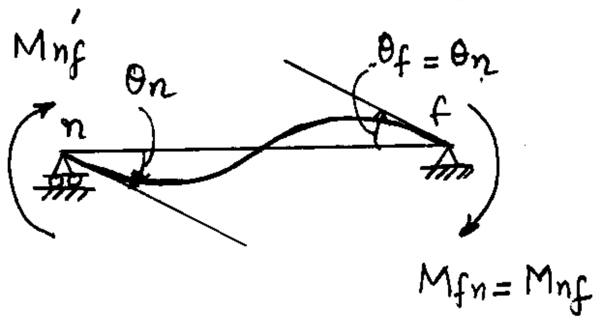
where

i, j = Far ends of members framing into joint b

D'_{bi} = Distribution Factors in which Modified Relative Stiffnesses have been used

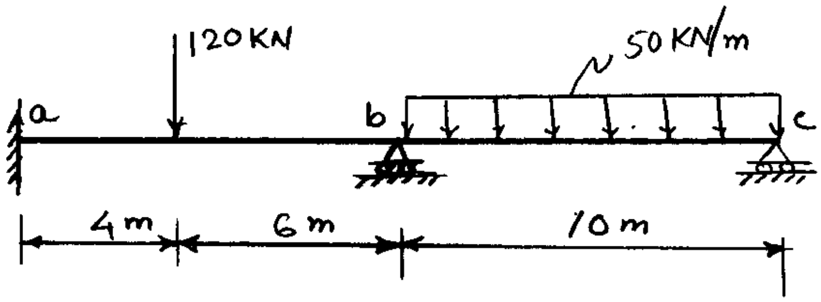
Modified Relative Stiffness for Exterior spans with Roller on Exterior End $= K^m_{nf} = \frac{1}{2} K_{nf}$. —— (11)

MODIFIED RELATIVE STIFFNESS FOR SELECT CASES

Loading Case	Stiffness M_{nf}'/θ_n	K_{nf}^{m}	Description of Condition
Normal	$4EK_{nf}$	—	
Simple Support	$3EK_{nf} = 4E(K_{nf}^{m})$	$\frac{2}{3}K_{nf}$	
Symmetric	$2EK_{nf} = 4E(K_{nf}^{m})$	$\frac{1}{2}K_{nf}$	
Antisymmetric	$6EK_{nf} = 4E(K_{nf})$	$\frac{3}{2}K_{nf}$	

Moment Distribution

Numerical Example using Modified Stiffness



Solve the above structure using modified moment distribution method

Stiffness and Relative Stiffness

$$K_{ab} = K_{ba} = \frac{I}{L} = \frac{I}{10} = K$$

$$K_{bc} = K_{cb} = \frac{I}{L} = \frac{I}{10} = K$$

$$K_{bc}^{m} = K_{cb}^{m} = \frac{3}{4} K_{bc} = \frac{3}{4} K$$

Distribution Factors

$$D_{ba} = \frac{K_{ab}}{K_{ab} + K_{bc}^{m}} = \frac{K}{K + \frac{3}{4}K} = 0.571$$

$$D_{bc} = \frac{K_{bc}^{m}}{K_{ba} + K_{bc}^{m}} = \frac{3/4\,K}{K + 3/4\,K} = 0.429$$

$$D_{cb} = \frac{K_{cb}}{K_{cb}} = \frac{K}{K} = 1.0$$

Fixed End Moments

$$FEM_{ab} = \frac{Pab^2}{L^2} = -\frac{120 \times 4 \times 6^2}{10^2} = -172.8 \text{ KN-m}$$

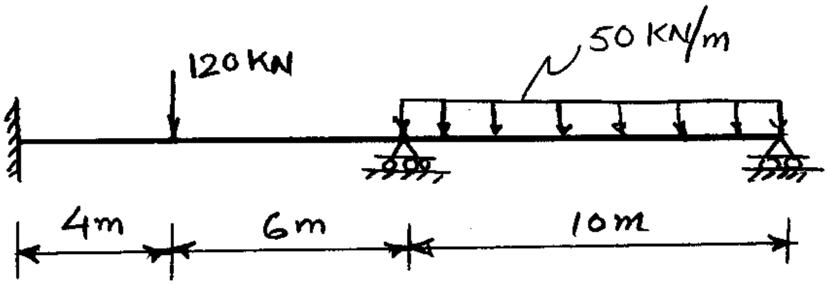
$$FEM_{ba} = \frac{Pa^2b}{L} = \frac{-120 \times 4^2 \times 6}{10^2} = +115.2 \text{ ''}$$

$$FEM_{bc} = -\frac{wl^2}{12} = -\frac{50 \times 10^2}{12} = -416.7 \text{ ''}$$

$$FEM_{cb} = +\frac{wl^2}{12} = \quad = +416.7 \text{ ''}$$

Moment Distribution

Numerical Example using modified Stiffness.



Dist Factor (D.F)	—	0.571	0.429	1.0
Carry Over Factor (C.O.F)	—	0.5	—	0.5
Fixed End Moments FEM	−172.8	+115.2	−416.7	+416.7
	+86.1 ←	+172.2	+129.3	
			−208.4 ←	−416.7
	+59.5 ←	+119.0	+89.4	
Final Moments	−27.2	+406.4	−406.4	0

→ Exact Solution Coverged to in 2 cycles.

ALTERNATE DISTRIBUTION

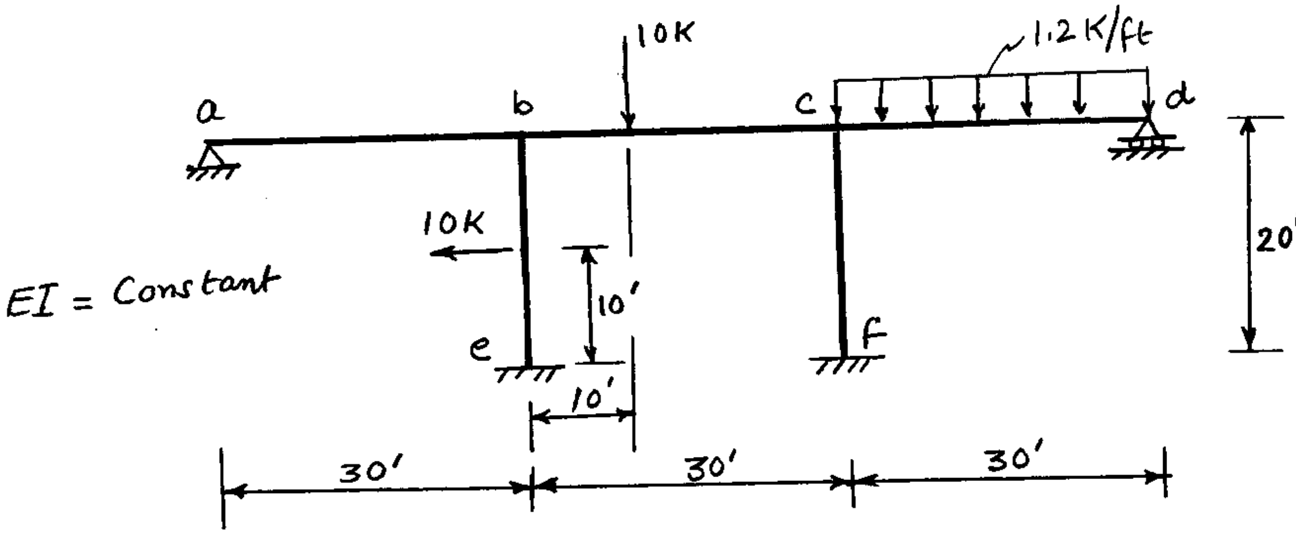
D.F	—	0.571	0.429	1.0
C.O	—	0.5	—	0.5
FEM	−172.8	+115.2	−416.7	+416.7
			−208.4 ←	−416.7
	+145.6 ←	+291.2	+218.7	
Final Moments	−27.2	+406.4	−406.4	0

→ Exact Solution Converged to in 1 cycle

Moment Distribution in Frames

Example Problem

Find the end moments for all the members of the frame shown below. The frame is restrained against sway at pt a.



Stiffnesses & Relative Stiffnesses

$K_{ab} = K_{ba} = K_{bc} = K_{cb} = K_{cd} = K_{dc} = \frac{I}{30} = K$

$K_{be} = K_{eb} = K_{cf} = K_{fc} = \frac{I}{20} = = 1.5K$

Modified Stiffnesses

$K^m_{ba} = \frac{3}{4} K_{ba} = 0.75K$

$K^m_{cd} = \frac{3}{4} K_{cd} = 0.75K$

Distribution Factors

$$D'_{bi} = \frac{K'_{bi}}{\sum_j K_{bi}}$$

$$D'_{ab} = \frac{K}{K} = 1.0$$

$$D_{ba} = \frac{0.75K}{0.75K + K + 1.5K} = 0.231$$

$$D'_{bc} = \frac{K}{(0.75 + 1 + 1.5)K} = 0.307$$

$$D'_{be} = \frac{1.5K}{(0.75 + 1 + 1.5)K} = 0.462$$

Moment Distribution in Frames

Example Problem

Fixed End Moments

$$FEM_{bc} = -\frac{Pab^2}{l^2} = -\frac{10 \times 10 \times 20^2}{30^2} = -44.4 \text{ K-ft}$$

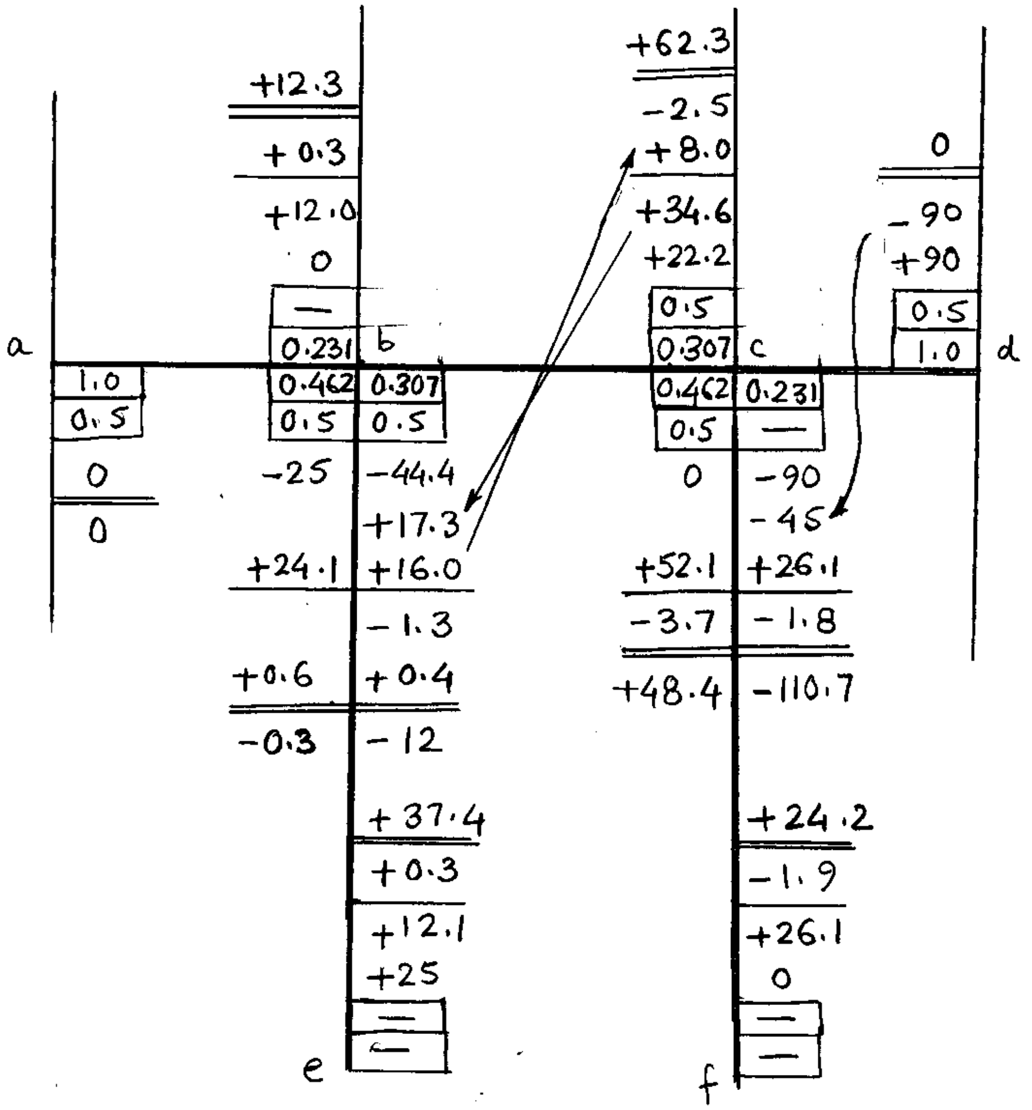
$$FEM_{cb} = -\frac{Pa^2b}{l^2} = \frac{10 \times 10^2 \times 20}{30^2} = +22.2 \text{ K-ft}$$

$$FEM_{cd} = -\frac{wl^2}{12} = -\frac{1.2 \times 30^2}{12} = -90$$

$$FEM_{dc} = \frac{wl^2}{12} = +90$$

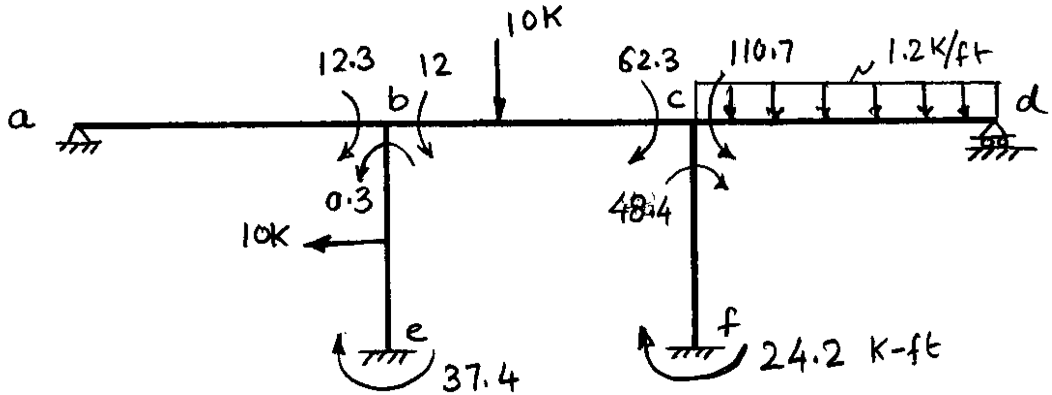
$$FEM_{be} = -\frac{Pab^2}{l^2} = -\frac{10 \times 10 \times 10^2}{20^2} = -25$$

$$FEM_{eb} = -\frac{Pa^2b}{l^2} = +25$$



Moment Distribution in Frames

Example Problem



END MOMENTS IN THE FRAME