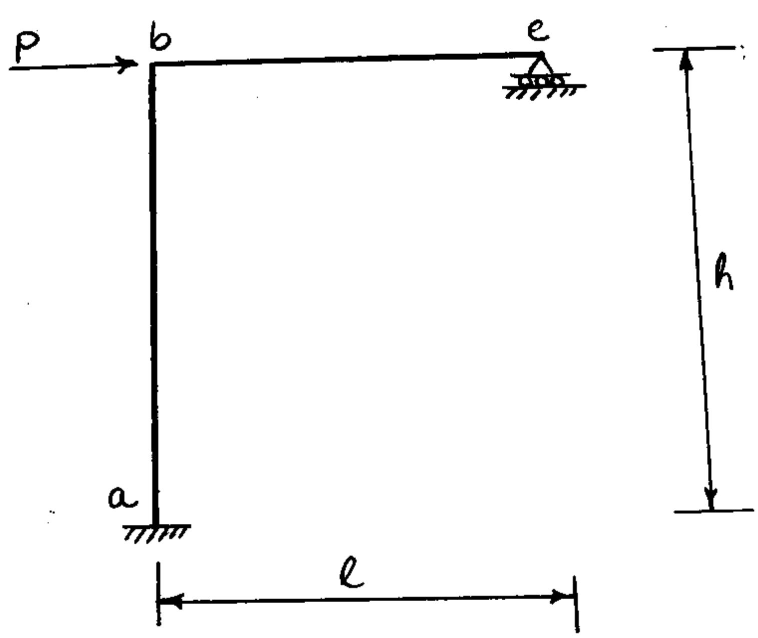
Moment Distribution Method

Frames Subjected to sway

- The Problem of analysis of beams subjected to settlements of supports is easy since the support settlements are prescribed.
- The analysis of frames subjected to sway is complicated by the fact that the amount of sway is not known apriori
- However, the frames subjected to sway can be analyzed by moment distribution method by invoking laws of equilibrium.
- The procedure is illustrated below:

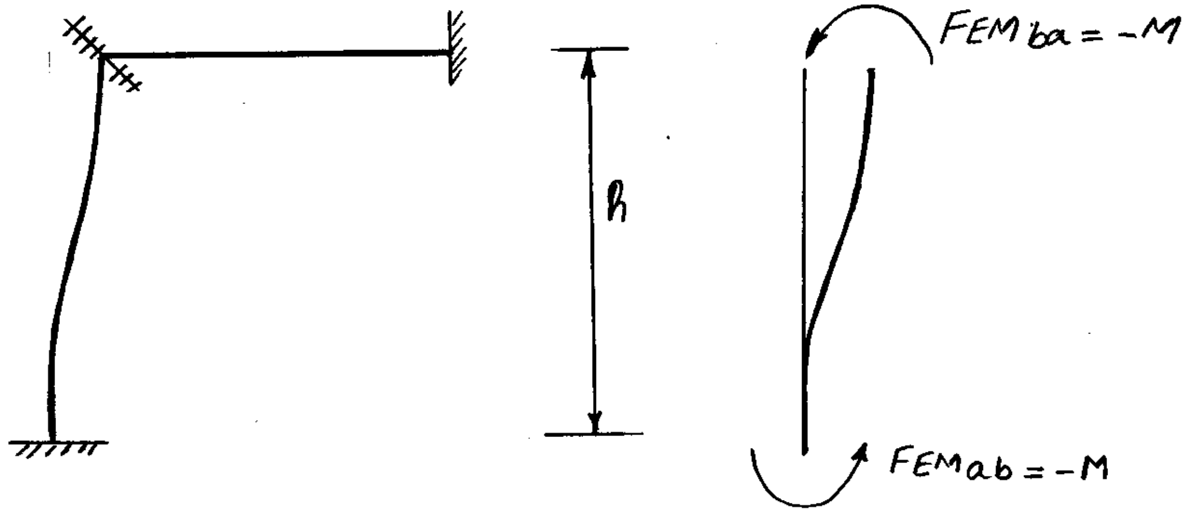
Consider the frame abc subjected to sway producing loads as shown below:



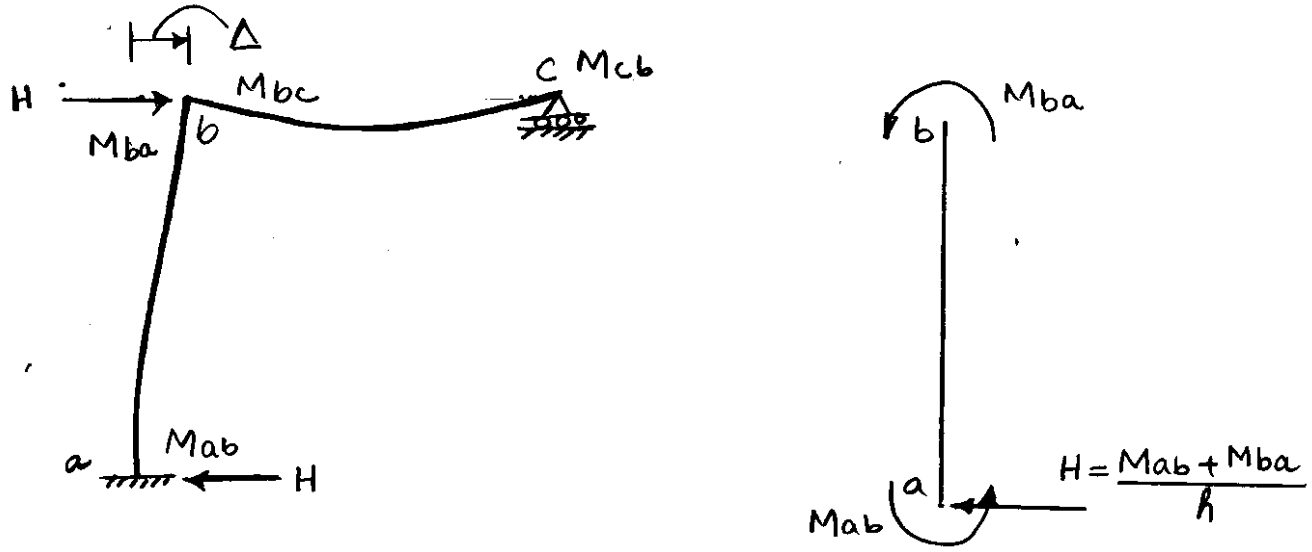
Moment Distribution Method

Frames Subjected to Sway

- A set of arbitrary fixed end moments is introduced in column ab



- Moment Distribution is then performed till an equilibrium solution is obtained as shown below:



- The free-body diagram of member ab suggests that there must be a base reaction H at end a. This base reaction must be equilibrated by application of corresponding force "H" at joint "b" of the structure

Moment Distribution Method

Frames subjected to Sway

- In general the base reaction "H" would not be equal to applied lateral load "P" i.e

$$H \neq P$$

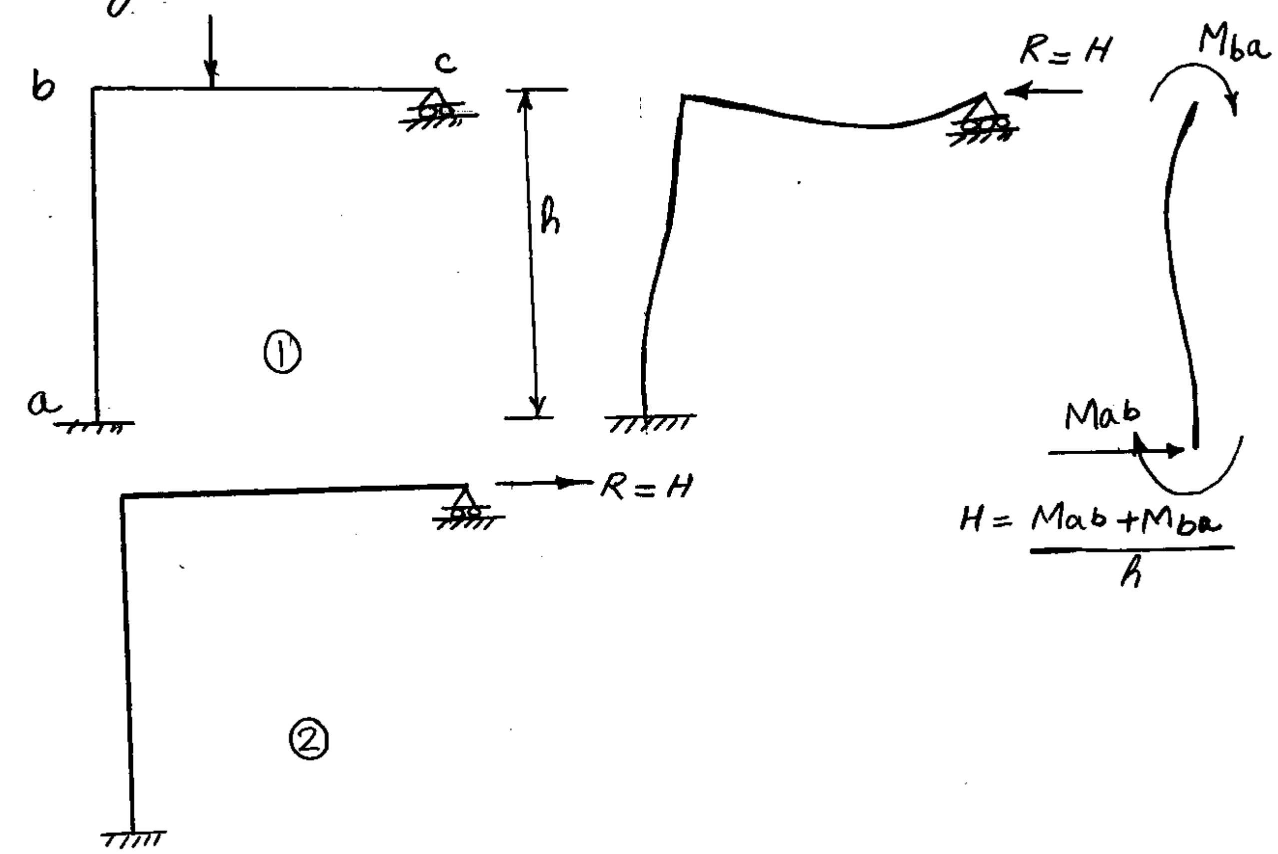
Then the solution of moments obtained corresponding to base reaction "H" needs to be multiplied by a factor "P/H" to obtain the correct solution corresponding to applied lateral load "P" i.e

$$\boxed{\text{Correct Soln corresponding to Lateral Load } P = \frac{P}{H} \times \text{Solution Corresponding to Base Reaction "H"}}$$

Moment Distribution Method

Frames subjected to sway

Case of no lateral sway causing loads



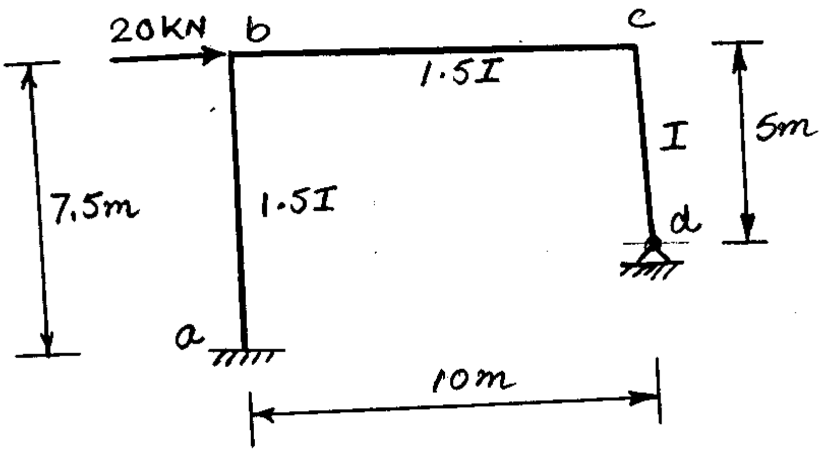
$$H = \frac{M_{ab} + M_{ba}}{h}$$

- Sway in frames can occur in the absence of lateral loading as well.. For example consider frame shown above.
- In this case solution is first obtained by moment distribution without considering sway as usual.
- Free body diagram of column "ab" shows that a base reaction "H" is required at end "a". This would require that a restraining force $R = -H$ be applied at joint "c" to equilibrate the structure
- However, since the boundary conditions at "c" do not allow application of this force, a force $= \vec{H}$ needs to be applied at joint "C", so that the reaction H at "a" vanishes. Correct solution is obtained by superimposing the two cases of applied loads and load "H"

Moment Distribution Method

Example Problem – Frame with Sway

Determine the end moments for each member and the support reactions for the frame shown below:



Stiffnesses & Relative Stiffnesses

$$K_{ab} = K_{ba} = \frac{1.5I}{7.5} = 0.2K = \bar{K}$$

$$K_{bc} = K_{bc} = \frac{1.5I}{10} = 0.15K = 0.75\bar{K}$$

$$K_{cd} = K_{dc} = \frac{I}{5} = 0.2K = \bar{K}$$

Modified Stiffnesses

$$K^{m}_{cd} = \frac{3}{4} K_{cd} = 0.75\bar{K}$$

Distribution Factors

$$D'_{bi} = \frac{K'_{bi}}{\sum\limits_{j} K'_{bj}}$$

At Joint b

$$D'_{ba} = \frac{K_{ba}}{K_{ba} + K_{bc}} = \frac{\bar{K}}{\bar{K} + 0.75\bar{K}} = 0.571$$

$$D_{bc} = \frac{K_{bc}}{K_{bc} + K_{ba}} = \frac{0.75\bar{K}}{\bar{K} + 0.75\bar{K}} = 0.429$$

Moment Distribution Method

Example Problem - Frame with Sway

At Joint c

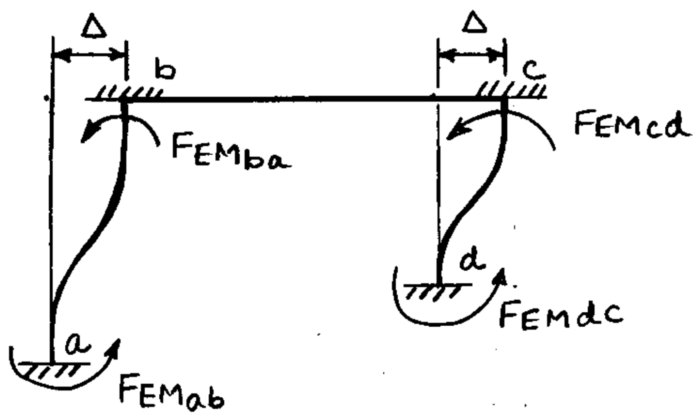
$$D'_{cb} = \frac{K_{cb}}{K_{cb} + K_{cd}^{(m)}} = \frac{0.75K}{0.75K + 0.75K} = 0.5$$

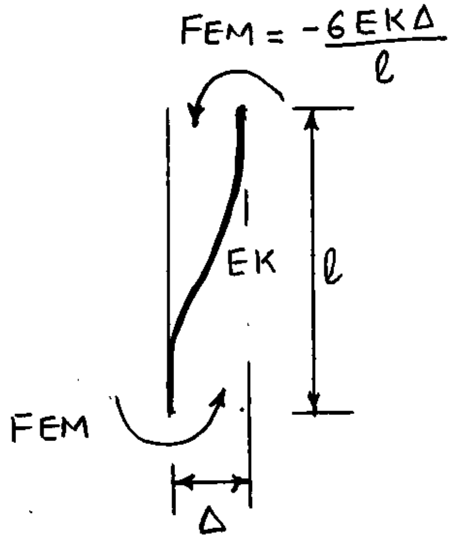
$$D'_{cd} = \frac{K_{cd}^{(m)}}{K_{cb} + K_{cd}^{(m)}} = \frac{0.75K}{0.75K + 0.75K} = 0.5$$

Moment Distribution Method

Example Problem - Frame with Sway

Fixed End Moments





For Displacement = Δ

$$FEM_{ab} = FEM_{ba} = -\frac{6EK_{ab}\Delta}{l_{ab}} = -6E\,\frac{1.5I/7.5}{7.5}\,\Delta$$

$$= -6EI(0.0267\Delta)$$

$$FEM_{cd} = FEM_{dc} = \frac{-6EK_{cd}\Delta}{l_{cd}} = -6E\,\frac{I/5}{5}\,\Delta$$

$$= -6EI(0.040\Delta)$$

The sway Δ is not known, but the relationship between the fixed end moments for columns is establised by the fixed end moment expressions above

If $FEM_{ab} = FEM_{ba} = -100\ KN\text{-}m$

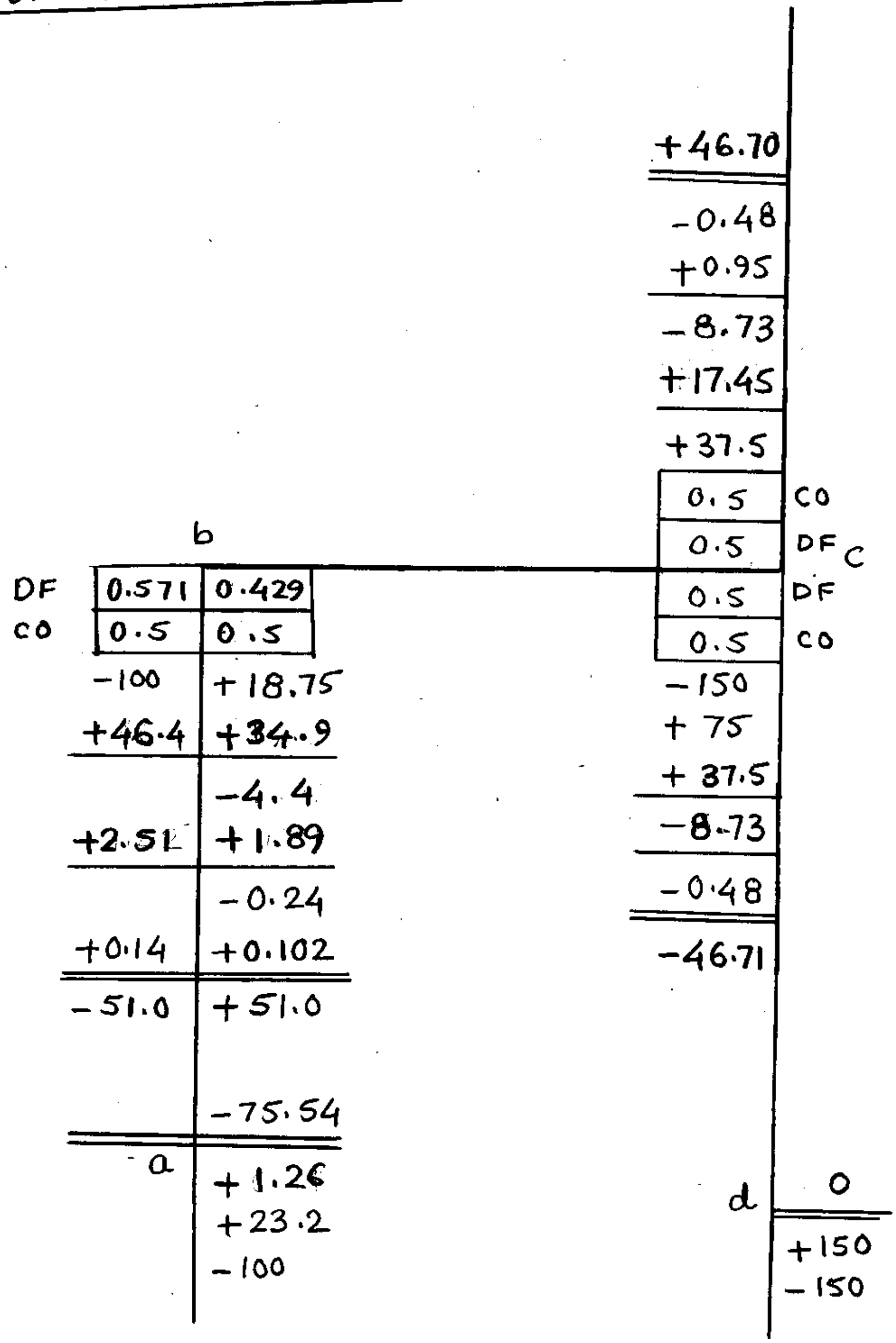
Then

$$FEM_{cd} = FEM_{dc} = -100 \times \left(\frac{-0.040}{-0.0267}\right)\frac{6EI\Delta}{6EI\Delta} = -150\ KN\text{-}m$$

Moment Distribution Method

Example Problem - Frame with Sway

Moment-Distribution



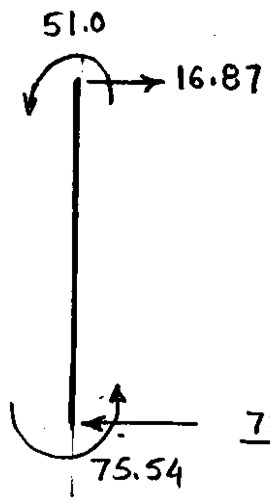
Moment Distribution Method

Example Problem - Frame with Sway

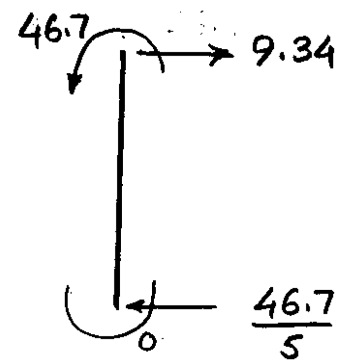
Equilibrium Considerations

Consider Free Body Diagrams of each member:

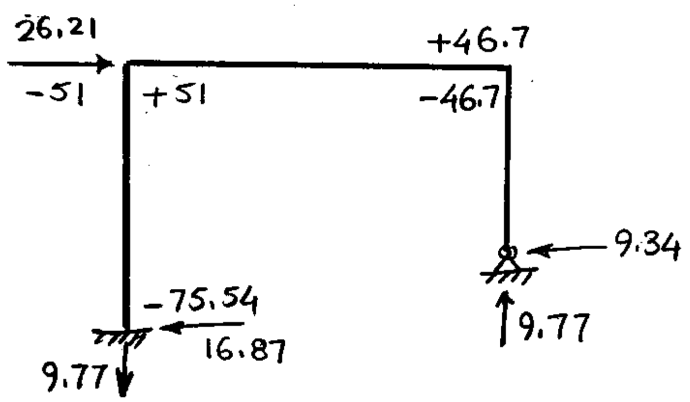
$$\frac{51.1+46.7}{10} = 9.78 \uparrow$$



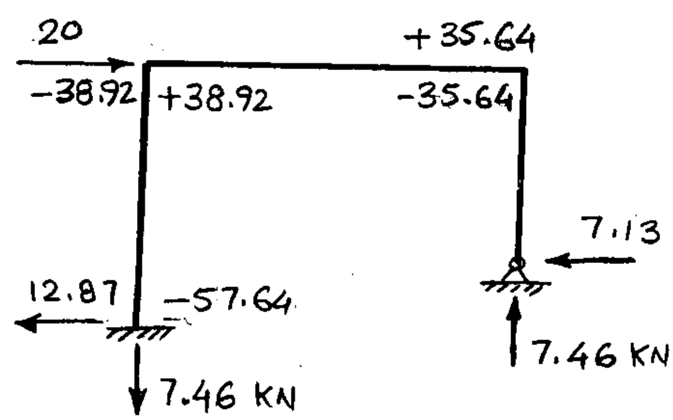
$$\frac{75.54+51.0}{7.5} = 16.87$$



$$\frac{46.7}{5} = 9.34$$



End Moments Corresponding to Lateral Force = 26.21



Final End Moments obtained by multiplication Factor 20/26.21

To Determine Moments Corresponding to Actual Lateral Force of 20 KN, Multiply the end moments by a factor $= \frac{20}{26.21}$

Flexibility Method of Analysis

Indeterminate structures can be solved using the "Flexibility Method". It is also called the "Compatibility Method"

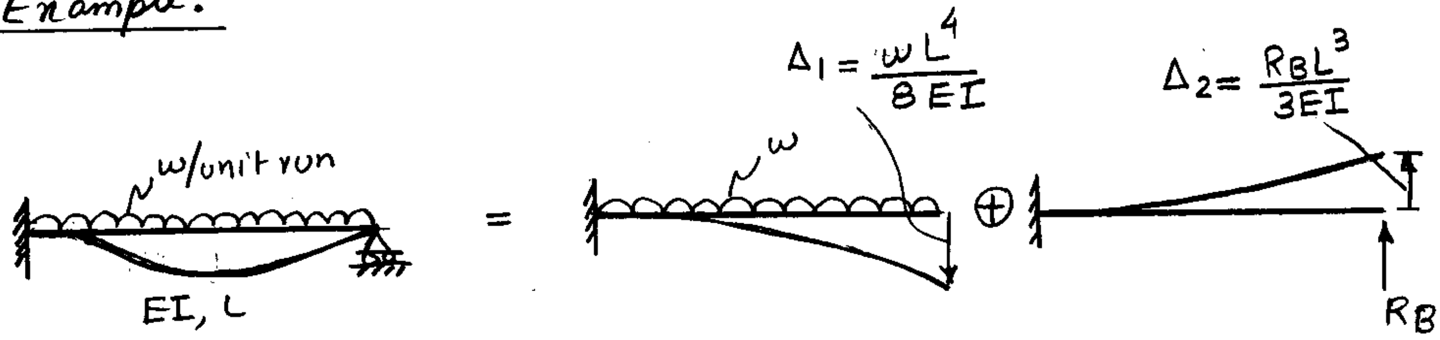
In Flexibility Method, Forces are taken as the unknown quantities and equations for forces are obtained by invoking Compatibility of displacements.

* The basic definition of "Flexibility" is that it is Deformation resulting from Unit Force

Among the classical Flexibility Methods or Force Methods are:

- Method of Consistent Deformations
- Method of Least Work.

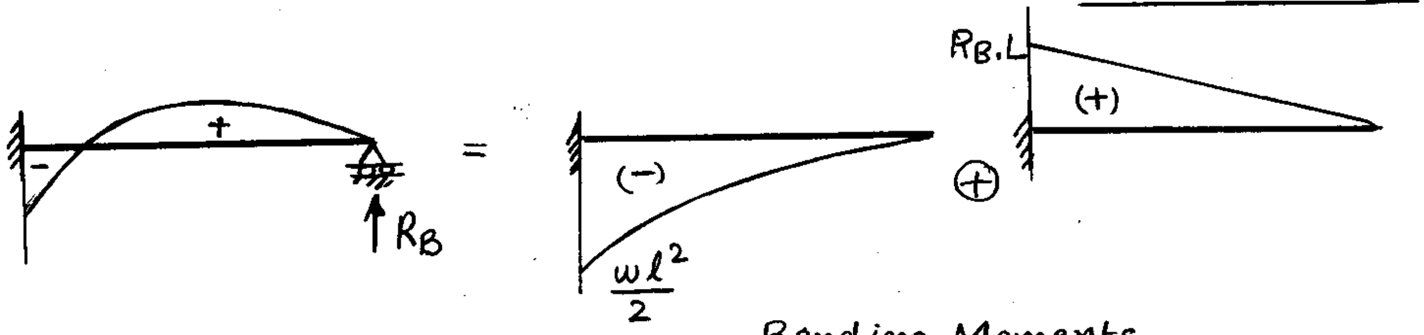
Example:



Indeterminate Structure

Primary Determinate Structure

Unknown Force Applied to Primary Structure



Bending Moments

Compatibility Condition:

$$\boxed{\begin{aligned} &\Delta_1 - \Delta_2 = 0 \\ \Rightarrow\ &\frac{wl^4}{8EI} - \frac{R_B l^3}{3EI} = 0 \end{aligned}} \Longrightarrow \boxed{R_B = \frac{3}{8} wl}$$

General Steps involved in Consistent Deformation Method

STEP 1 : Determine the "Degree of Indeterminancy" of the structure and convert the structure into a suitable Determinate Primary Structure.

Identify and impose the released forces/ Redundants.

STEP 2 : Compute displacements of the Primary Structure at the locations of Redundants due to applied loading only

STEP 3 : Compute displacements in the Primary Structure in the direction of Redundants due to variable value of Redundants

STEP 4 : Write expressions for total displacements at the Redundants locations interms of displacements of the Primary Structure due to applied loading and the variable Redundant loadings.

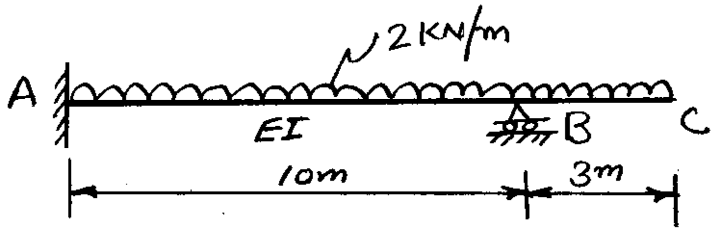
STEP 5 : Solve the displacement compatibility equations to solve for Redundant Forces.

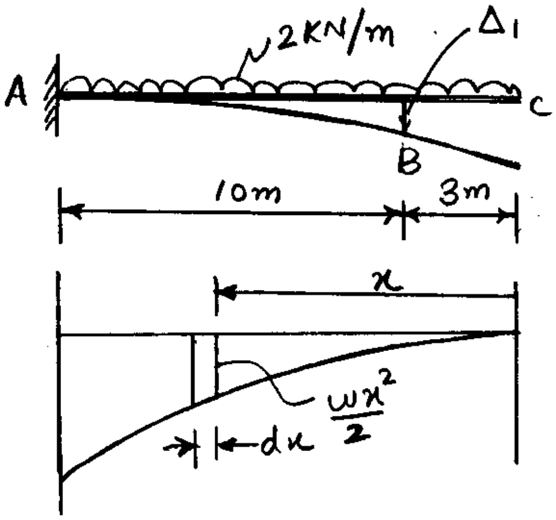
STEP 6 : Superimpose the effects of the Redundants on the Primary Structure to get Final Forces, Bending Moments and Shear Forces.

Flexibility Method of Analysis

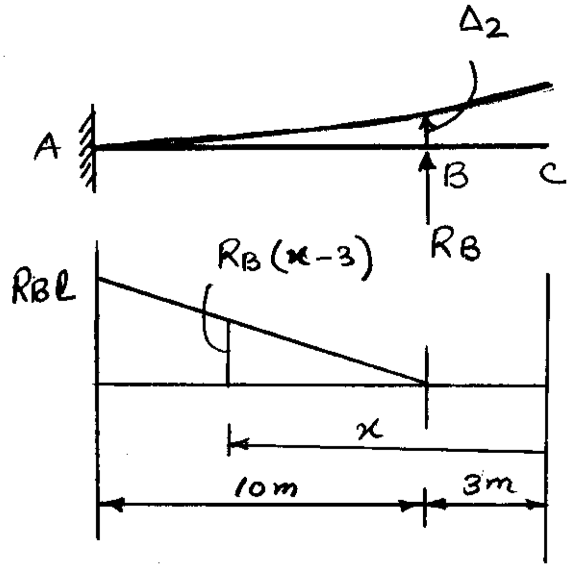
Example Problem

Analyze the propped cantilever shown below using method of consistent deformations.





Primary Structure & Loading



Primary Structure & Redundants Loading

Δ_1 = Deflection at Pt B in Primary Structure

$$= \int_3^{13} \frac{M}{EI} \cdot x \, dx = \int_3^{13} \frac{wx^2}{2EI}(x-3)\, dx$$

$$= \frac{w}{2EI}\int_3^{13} (x^3 - 3x^2)\, dx = \frac{w}{2EI}\left|\frac{x^4}{4} - x^3\right|_3^{13}$$

$$= \frac{w}{2EI}\left[\left(\frac{13^4 - 3^4}{4}\right) - \left(13^3 - 3^3\right)\right] = \frac{4950\, w}{2EI}$$

$$\Delta_1 = \frac{4950 \times 2}{2\, EI} = \frac{4950}{EI}$$

Example Problem

Δ_2 = Deflection at Pt B in Primary Structure due to Redundant RB

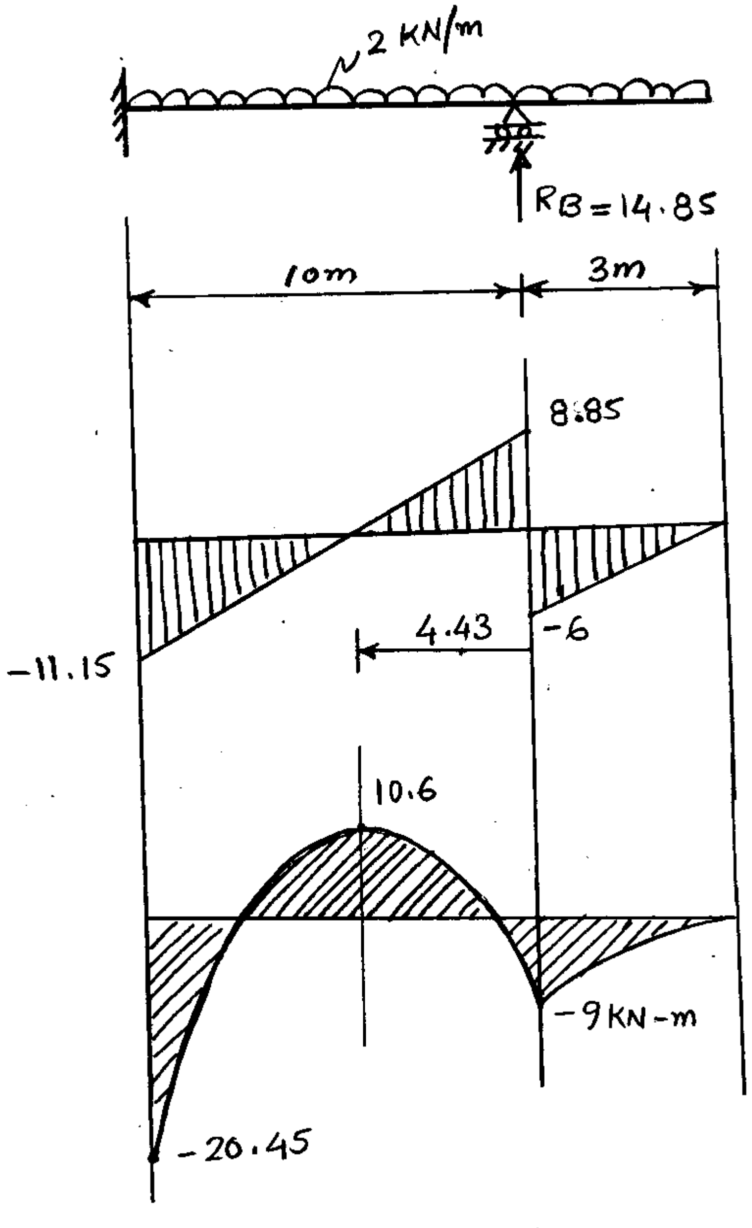
$$\Delta_2 = \frac{RB \times 10^3}{3EI} = \frac{1000\,RB}{3EI}$$

Compatibility Equation

$$\Delta_1 - \Delta_2 = 0$$

$$\frac{4950}{EI} - \frac{1000\,RB}{3EI} \Rightarrow$$

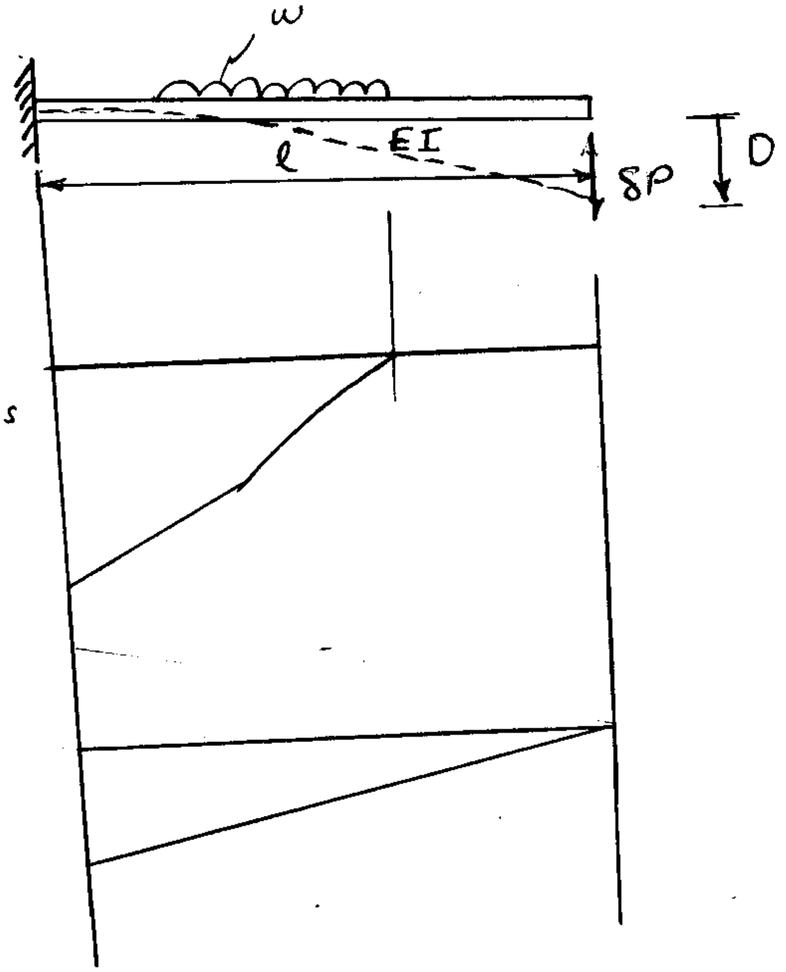
$$\boxed{RB = \frac{3 \times 4950}{1000} \qquad RB = 14.85\ KN}$$



SHEAR FORCE

BENDING MOMENT

Virtual Work Expression For Beams & Frames due to Bending



Bending Moment Due to Actual Loads

Bending Moment due to Virtual Loads

Consider the Beam above acted upon by actual load "w" and a Virtual unit load δP at the tip of the beam. The Virtual Work expression for such a system can be written as:

Virtual Forces/Stresses

$$\sum (\delta P)_i \, D_i = \int_{Vol} (\delta \sigma_P)\, \varepsilon_D \, d\,vol \qquad \text{——— (1)}$$

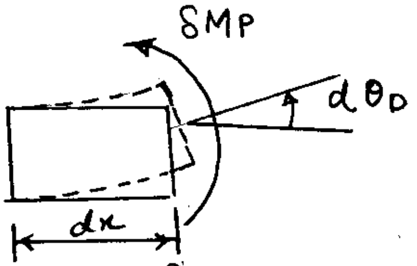
Actual Displacements

External Virtual Work = Internal Virtual Work

Virtual Work Expression For Beams & Frames Due to Bending

$$\text{Internal Virtual Work} = \int_{Vol} (\delta\sigma_P)\, \varepsilon_D \, dVol$$

$$= \sum_{j=1}^{m} \left(\int_{l} \delta M_P \, d\theta_D \right)_j \qquad \text{--- (2)}$$



Virtual Work Done in a Differential Element $= (\delta M_P)\, d\theta_D$

$$\text{Now} \quad d\theta_D = \frac{M_D}{EI} \cdot dx \qquad \text{--- (3)}$$

From (1), (2) and (3) we get:

Virtual System

$$\sum (\delta P)_i \, D_i = \sum_{j=1}^{m} \int_{l} \left(\delta M_P \cdot \frac{M_D}{EI} \, dx \right)_j \qquad \text{--- (4)}$$

Actual System

If δP_i = single unit load

Then (4) reduces to

$$\boxed{D_i = \sum_{j=1}^{m} \int_{l} \left(\frac{M_V M_D}{EI} \right)_j} \qquad \text{--- (5)}$$

where M_V = Virtual Moment due to unit Dummy Load

M_D = Moments due to actual Loads