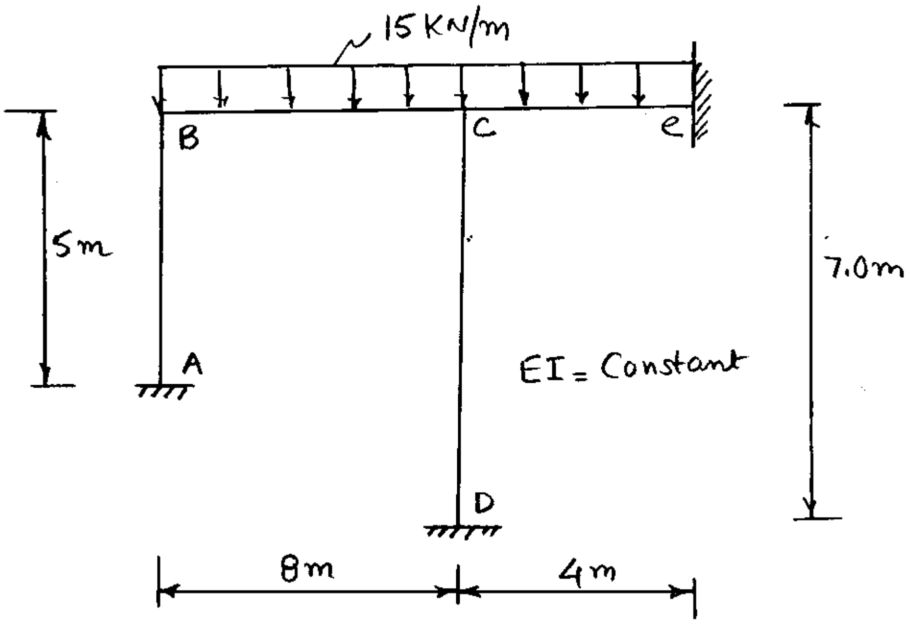
Advanced Structural Analysis
Midterm Exam - Solution

QNo 1 Determine the moments in the portal Frame using moment Distribution method.



Stiffnesses & Relative Stiffnesses

$K_{ab} = K_{ba} = \frac{I}{5} = 0.2K$

$K_{bc} = K_{cb} = \frac{I}{8} = 0.125K$

$K_{cd} = K_{dc} = \frac{I}{7} = 0.143K$

$K_{ce} = K_{ec} = \frac{I}{4} = 0.25K$

Distribution Factors

$D_{ba} = \frac{0.2}{0.2+0.125} = 0.615$

$D_{bc} = \frac{0.125}{0.2+0.125} = 0.385$

$D_{cb} = \frac{0.125}{0.125+0.25+0.143} = \frac{0.125}{0.518} = 0.241$

$D_{ce} = \frac{0.25}{0.518} = 0.483$

$D_{cd} = \frac{0.143}{0.518} = 0.276$

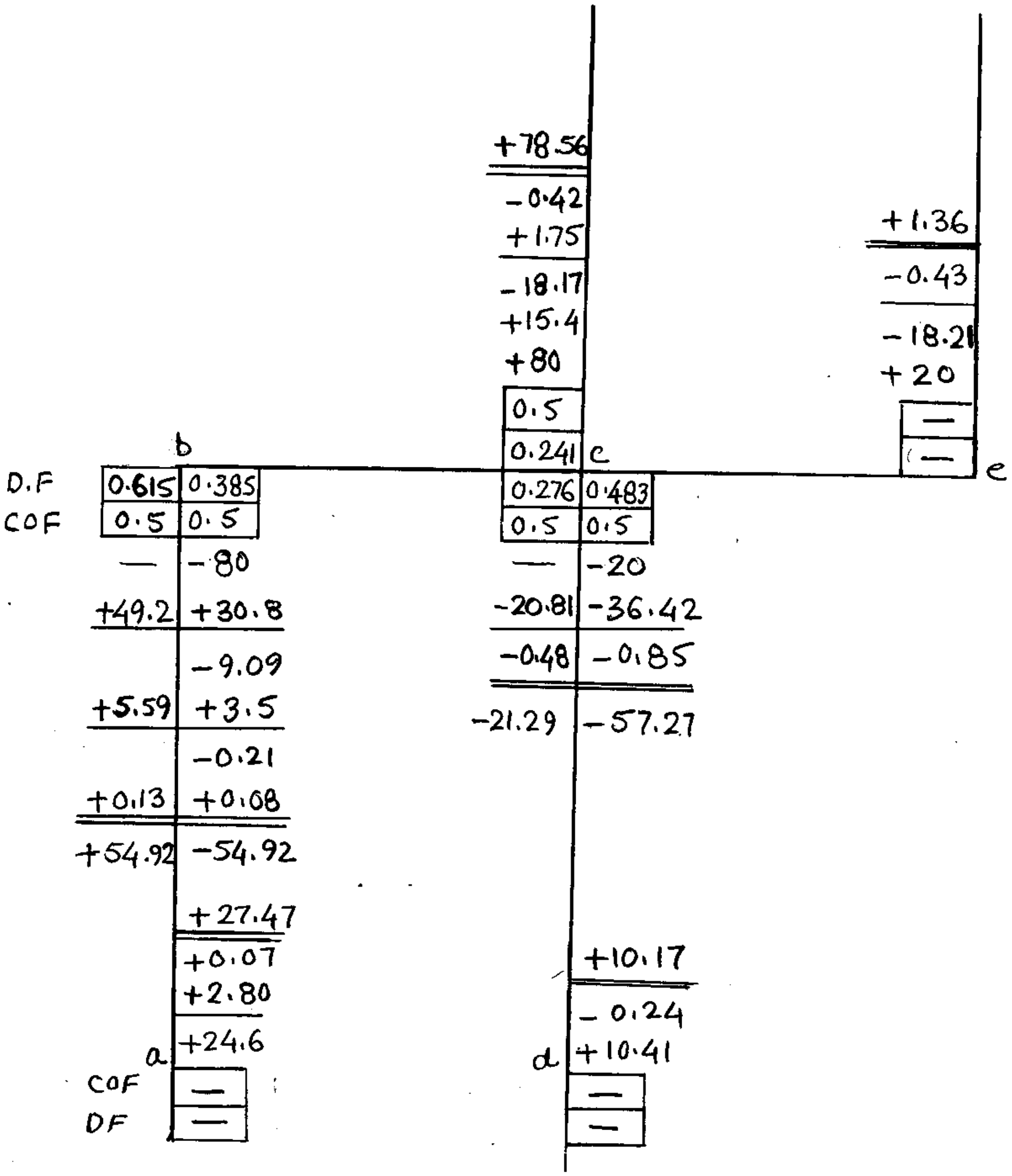
- Fixed End Moments Clock-wise moment +ive

$$FEM_{bc} = -\frac{wl^2}{12} = \frac{15 \times 8^2}{12} = -80 \text{ KN-m}$$

$$FEM_{cb} = +\frac{wl^2}{12} = +80 \text{ KN-m}$$

$$FEM_{ce} = -\frac{wl^2}{12} = \frac{15 \times 4^2}{12} = -20 \text{ KN-m}$$

$$FEM_{ec} = +\frac{wl^2}{12} = +20 \text{ KN-m}$$



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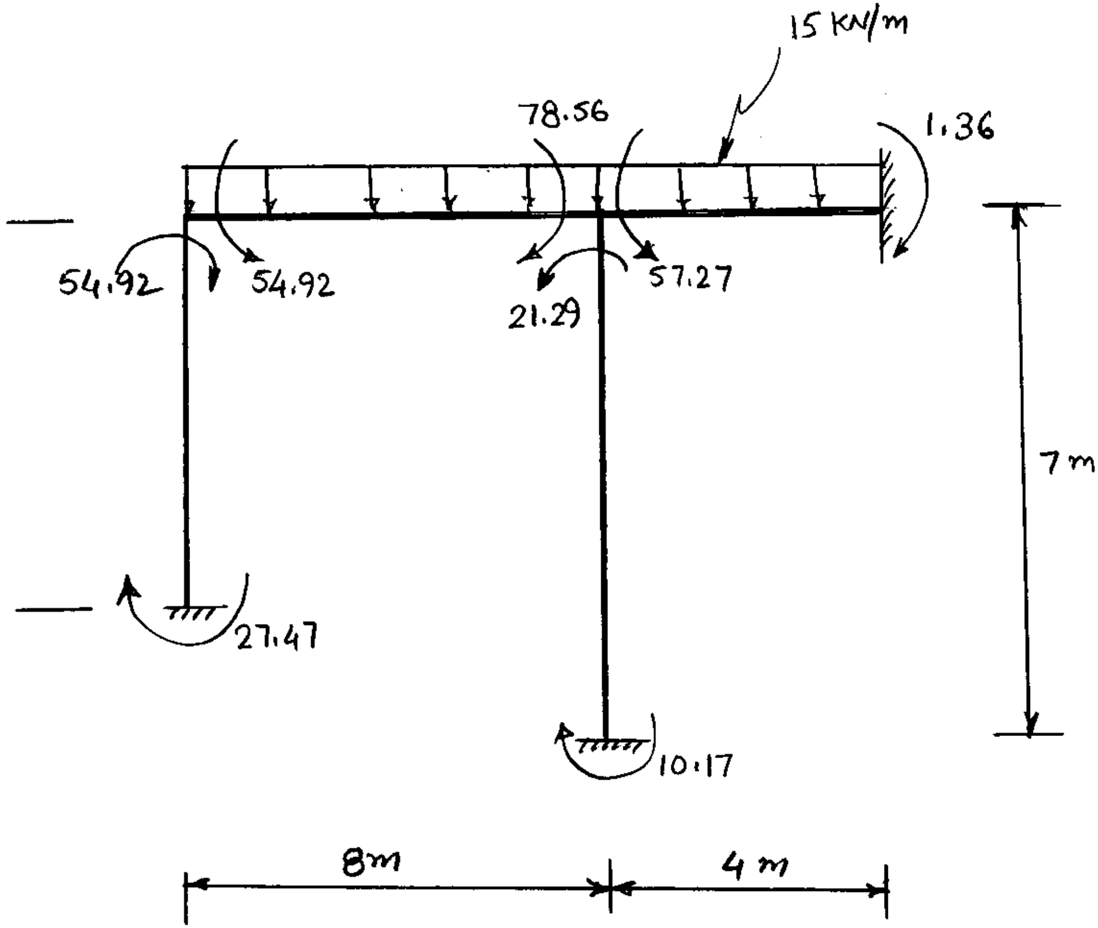
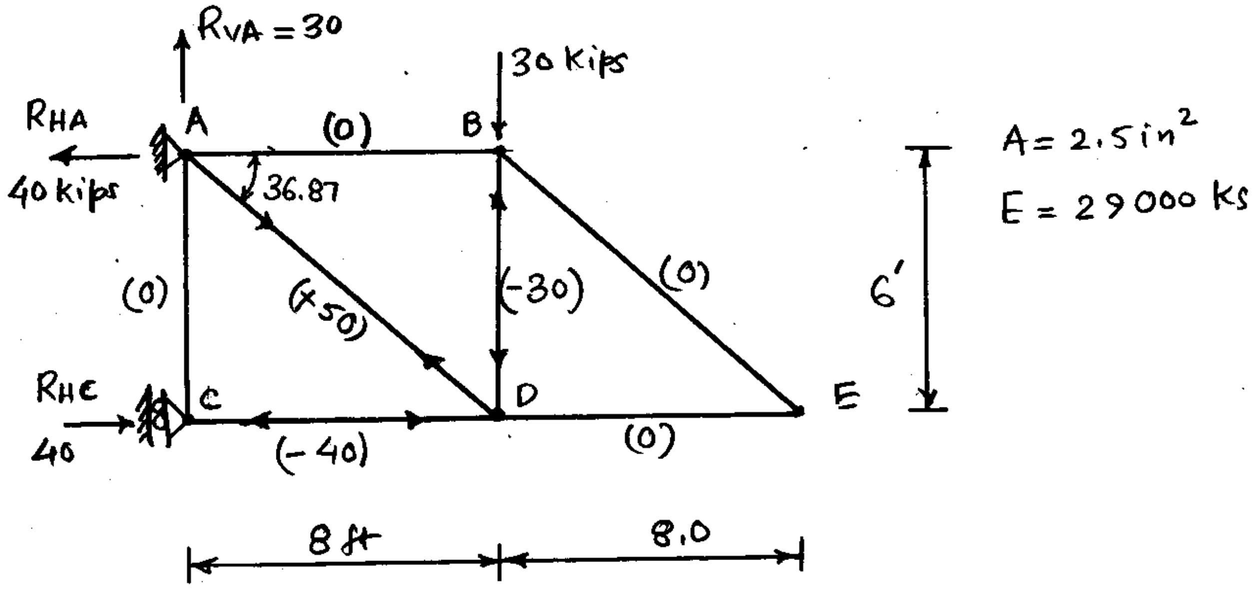


END MOMENTS

- QNo. 2 Find the vertical Deflection at pt E in the truss below using method of virtual work.



Determine Reactions

By observation $R_{VA} = 30$ Kips.

Taking moments @ A

$$30 \times 8 - R_{HC} \times 6 = 0 \Rightarrow R_{HC} = \frac{30 \times 8}{6} = 40 \text{ Kips} \rightarrow$$

$$\Rightarrow R_{HA} = -40 \text{ Kips} \leftarrow$$

Member Forces

$F_{CD} = -40$ kips (Comp)

By observation forces in members DE, BE $= 0$ as no nodal Force @ E

$F_{DE} = F_{BE} = 0$

Also,

$F_{AC} = 0$ as no vertical reaction @ C

$$F_{AD} \sin 36.87 = R_{VA} = 30 \Rightarrow F_{AD} = \frac{30}{\sin 36.87} = +50 \text{ (Tons)}$$

No. 2

Consider joint D

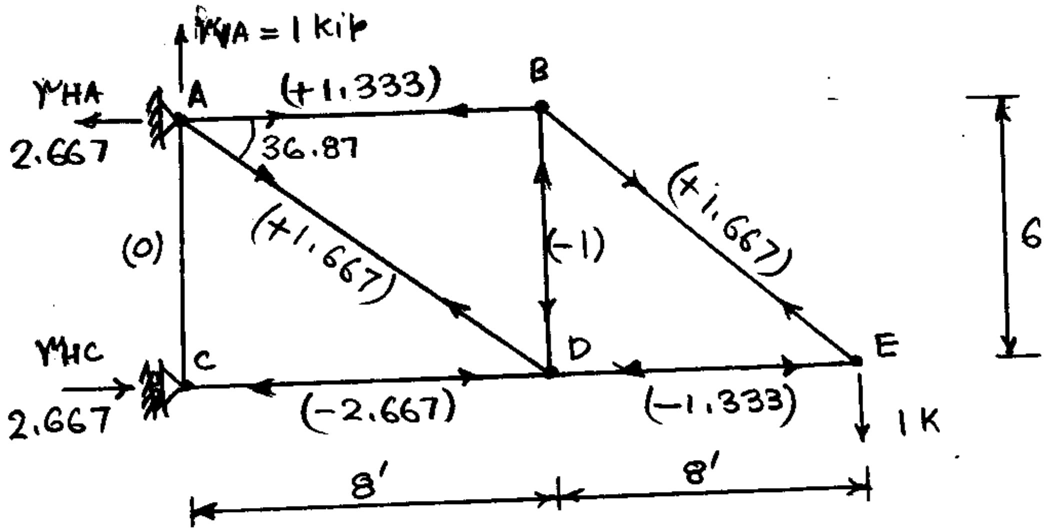
$F_{AC} \sin 36.87 + F_{BD} = 0 \Rightarrow F_{BD} = -F_{AC} \sin 36.87$

$F_{BD} = -50 \sin 36.87$

$F_{BD} = -30$ kip (Comp)

Then The remaining Force FAB has to be = 0 if equilibrium @ joint B is considered.

Forces in structure with Unit Load Applied



$r_{VA} = 1$ kip ↑

Moment @ A

$1 \times 16 - r_{HC} \times 6 = 0 \Rightarrow r_{HC} = \frac{16}{6} = 2.667$

$\Rightarrow r_{HA} = 2.667$

$f_{CD} = -2.667$ Kips (Comp)

f_{AC}

$f_{ad} \sin 36.87 = r_{VA} = 1 \Rightarrow f_{ad} = \frac{1}{\sin 36.87} = +1.667$ (Tens)

@ Joint A

$f_{AB} - r_{HA} + f_{ad} \cos 36.87 = 0$

$\Rightarrow f_{AB} = 2.667 - 1.667 \cos 36.87 = +1.333$ Kips (Tens)

Midterm Exam

QNo 2

At Joint E

$$f_{BE} \sin 36.87 - 1 = 0 \quad \Rightarrow \quad f_{BE} = \frac{1}{\sin 36.87} = +1.667 \text{ (Tens)}$$

$$f_{DE} + f_{BE} \cos 36.87 = 0 \quad \Rightarrow \quad f_{DE} = -1.667 \cos 36.87$$

$$f_{DE} = -1.333 \text{ Kips (Comp)}$$

@ Joint B

$$f_{BD} + f_{BE} \sin 36.87 = 0 \quad \Rightarrow \quad f_{BD} = -1.667 \sin 36.87$$

$$f_{BD} = -1.0 \text{ Kips (Comp)}$$

Apply Virtual Work Principle

$$1.\,\Delta_{VE} = \sum_{j=1}^{m} \frac{F.f.L}{AE} \quad ,$$

F = Primary Structure Forces

f = Forces due to unit load.

Member	F (Kibs)	L (ft)	f Kip	FfL
AB	0	8	1.333	—
AC	0	6	0	—
AD	50	10	1.667	833.5
CD	−40	8	−2.667	853.44
BD	−30	6	−1.0	180.0
BE	0	10	+1.667	—
DE	0	8	−1.333	—
			Σ	1866.94

$$1.\,\Delta_{VE} = \sum_{j=1}^{m} \frac{FfL}{AE} = \frac{1866.94}{\left(\frac{2.5 \text{ in}^2}{144}\right)(29000 \text{ ksi} \times 144)}$$

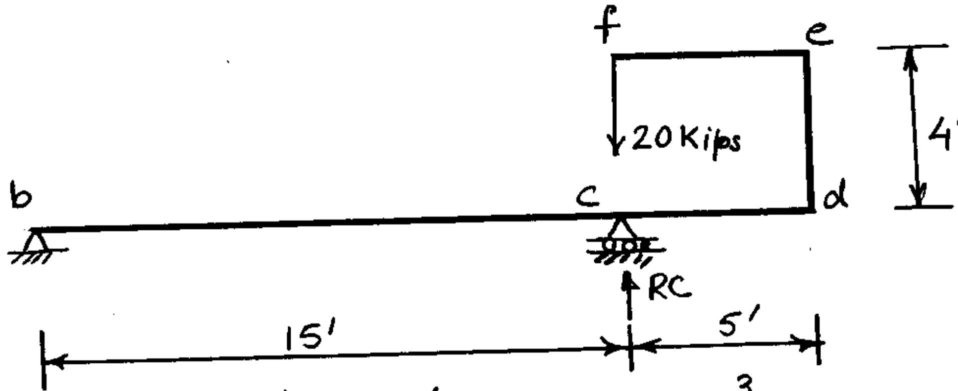
$$= \frac{1866.94}{2.5 \times 29000} = 0.02575 \text{ ft} \downarrow$$

$$= 0.309 \sim 0.31 \text{ in} \downarrow$$

Answer.

Mid term Exam

QNo 3 Calculate Vertical deflection at pt. f of the beam shown below using method of virtual work.



$I = 1000\ in^4$, $E = 29 \times 10^3$ KSi

Reactions from Global Equilibrium

Taking moment @ pt b

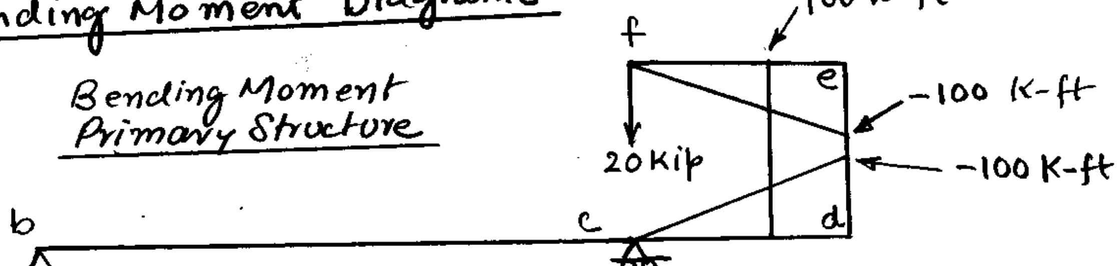
$20 \times 15 - R_C \times 15 = 0$

$\Rightarrow \quad R_C = 20$ kips ↑

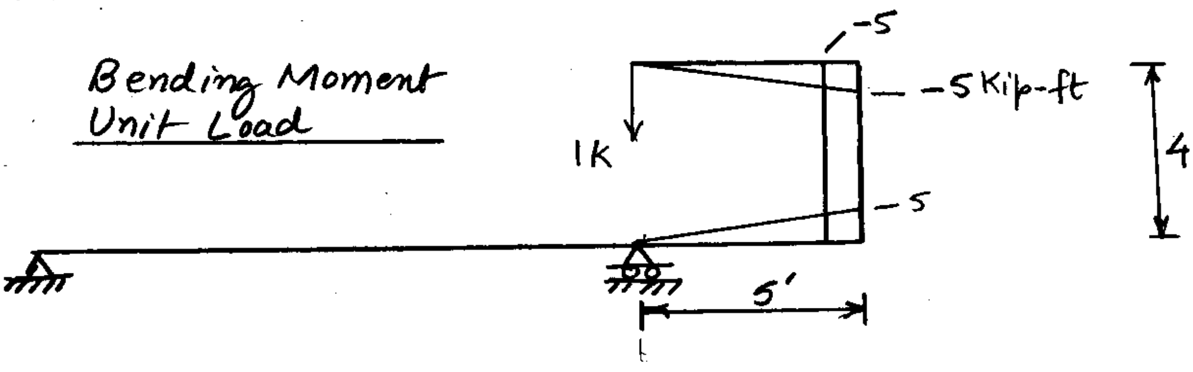
$\Rightarrow \quad R_b = 0$

Bending Moment Diagrams

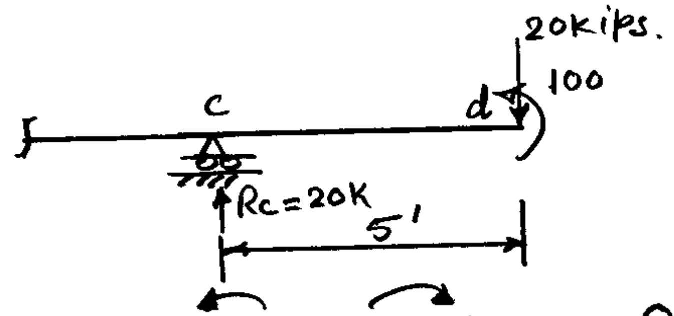
Bending Moment Primary Structure



Bending Moment Unit Load



Bending moment @ pt C



Moment @ pt. C $= 100 - 20 \times 5 = 0$

Virtual Work Expression

$$1.\ \Delta_{fV} = \sum_{j} \int_{l} \frac{Mm}{EI}\, dl$$

$$= \int_0^5 \frac{20x \cdot x}{EI}\, dx + \int_0^4 \frac{(-100)(-5)}{EI}\, dx + \int_0^5 \frac{20x \cdot x}{EI}\, dx$$

$$= 2\int_0^5 \frac{20x^2}{EI} + \int_0^4 \frac{500}{EI}\, dx$$

$$= \frac{1}{EI}\left\{\left[40\frac{x^3}{3}\right]_0^5 + \left[500x\right]_0^4\right\}$$

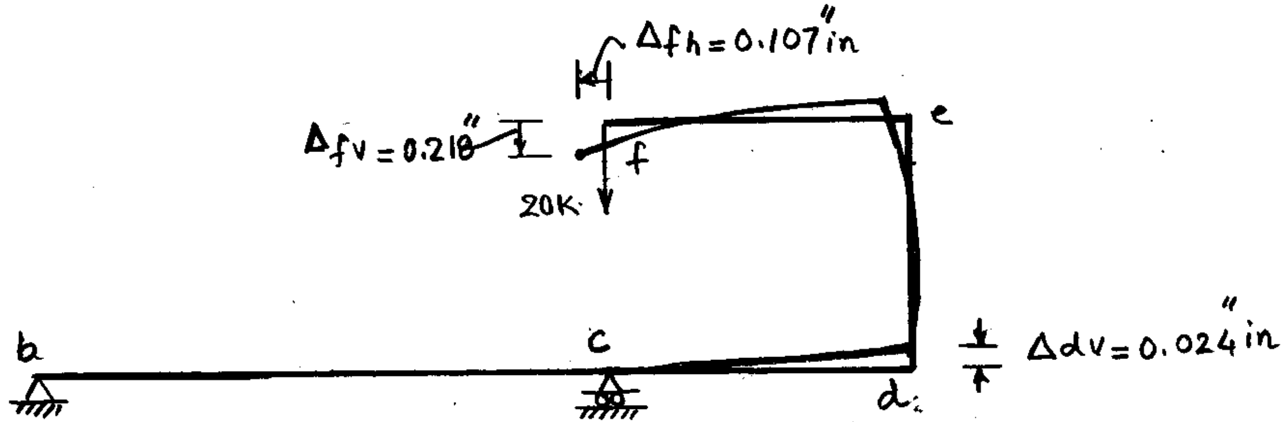
$$= \frac{1}{EI}\left\{\frac{40}{3}\times 5^3 + 500\times 4\right\} = \frac{3666.67}{EI}\ \text{kip-ft}$$

$$I = \frac{1000}{(12)^4}\ ft^4, \quad E = 29000 \times 12^2\ ft^2$$

$$1.\ \Delta_{fV} = \frac{3666.67 \times 12^2}{29000 \times 1000} = 0.0182\ ft = 0.218\ in \downarrow$$

Midterm Exam

Q No. 3



Deformed Shape